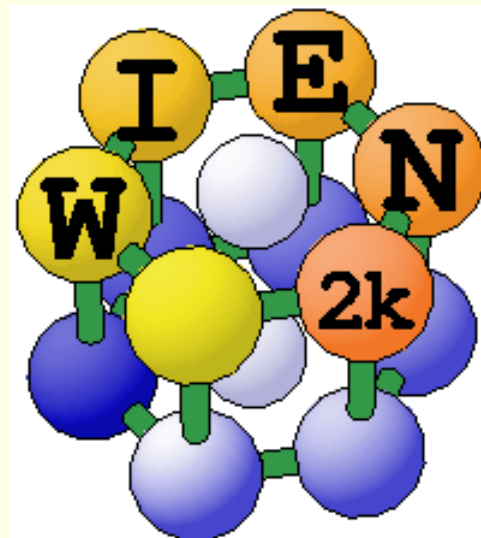


Hyperfine interactions

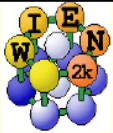
Karlheinz Schwarz

Institute of Materials Chemistry

TU Wien



Some slides were provided by Stefaan Cottenier (Gent)



Kohn-Sham equations



LDA, GGA

$$E = T_o[\rho] + \int V_{ext} \rho(\vec{r}) d\vec{r} + \frac{1}{2} \int \frac{\rho(\vec{r}) \rho(\vec{r}')}{|\vec{r}' - \vec{r}|} d\vec{r} d\vec{r}' + E_{xc}[\rho]$$

1-electron equations (Kohn Sham)

vary ρ

$$\left\{ -\frac{1}{2} \nabla^2 + V_{ext}(\vec{r}) + V_C(\rho(\vec{r})) + V_{xc}(\rho(\vec{r})) \right\} \Phi_i(\vec{r}) = \varepsilon_i \Phi_i(\vec{r})$$

$$-Z/r$$

$$\int \frac{\rho(\vec{r}')}{|\vec{r}' - \vec{r}|} d\vec{r}'$$

$$\frac{\partial E_{xc}(\rho)}{\partial \rho}$$

$$\rho(\vec{r}) = \sum_{\varepsilon_i \leq E_F} |\Phi_i|^2$$

$$E_{xc}^{LDA} \propto \int \rho(r) \varepsilon_{xc}^{hom}[\rho(r)] dr$$

$$E_{xc}^{GGA} \propto \int \rho(r) F[\rho(r), \nabla \rho(r)] dr$$

LDA

treats both, exchange and correlation effects,

GGA

but approximately

New (better ?) functionals are still an active field of research



Kohn-Sham equations



$$E = T_o[\rho] - \int V_{ext} \rho(\vec{r}) d\vec{r} - \frac{1}{2} \int$$

nuclear point charges
interacting with
electron charge distribution

1-electron equations (Kohn Sham)

vary ρ

$$\left\{ -\frac{1}{2} \nabla^2 + V_{ext}(\vec{r}) + V_C(\rho(\vec{r})) + V_{xc}(\rho(\vec{r})) \right\} \Phi_i(\vec{r}) = \varepsilon_i \Phi_i(\vec{r})$$

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$$\int \frac{\rho(\vec{r}')}{|\vec{r}' - \vec{r}|} d\vec{r}'$$

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LDA } treats both,
exchange and correlation effects,
GGA } but approximately

New (better ?) functionals are still an active field of research

Definition :

hyperfine interaction

=

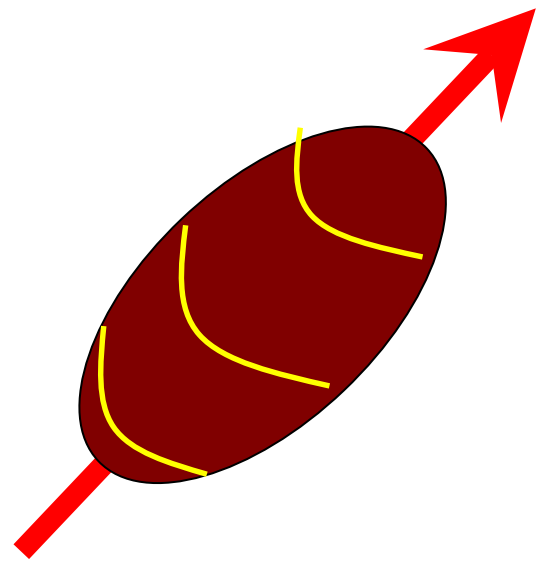
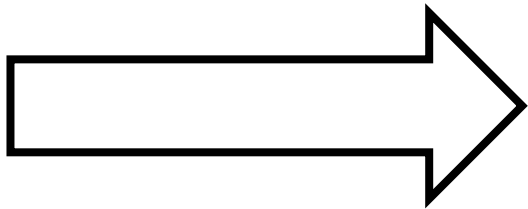
all aspects of the
nucleus-electron interaction

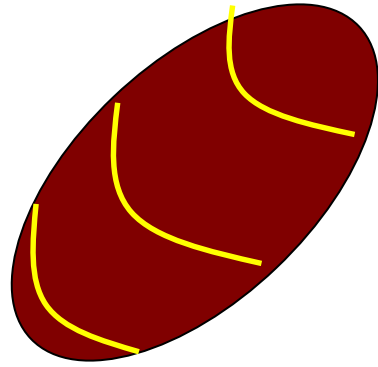
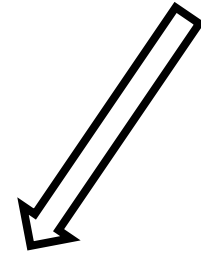
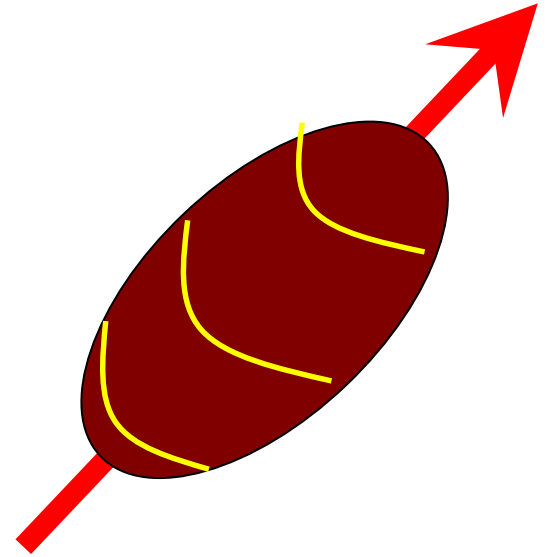
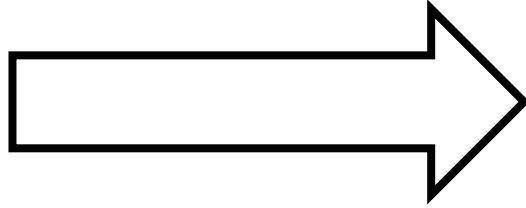
which go **beyond**

an electric **point charge** for a nucleus.



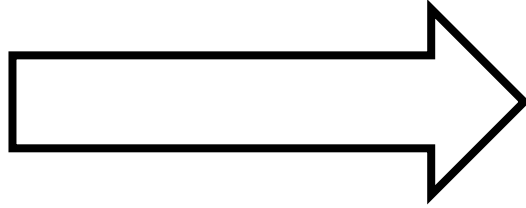
electric
point
charge



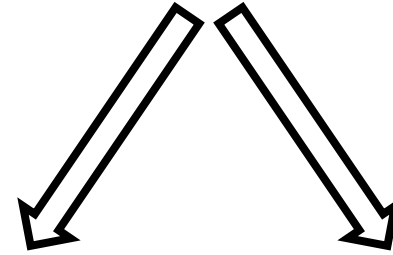
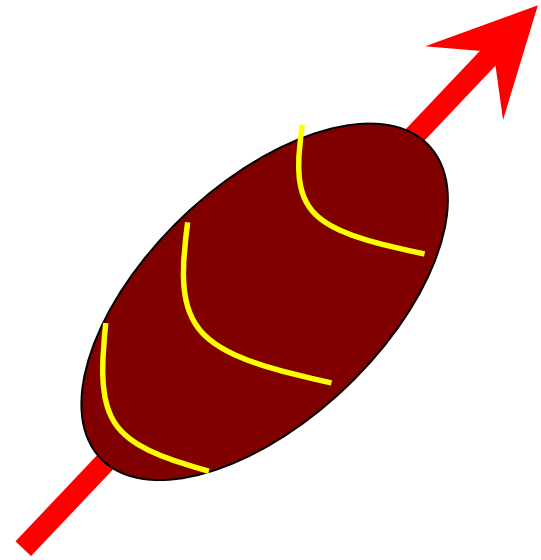


electric
~~point~~
charge

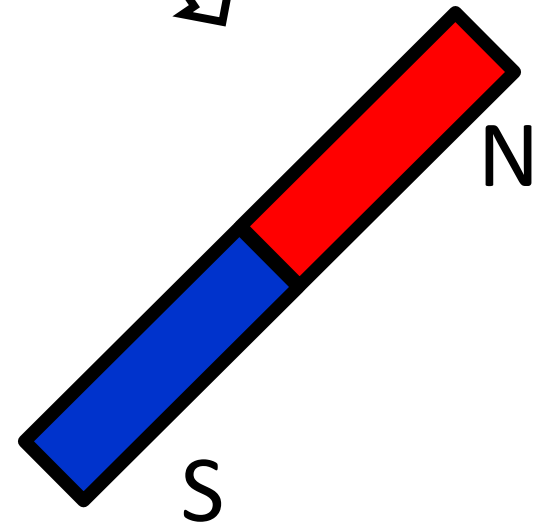
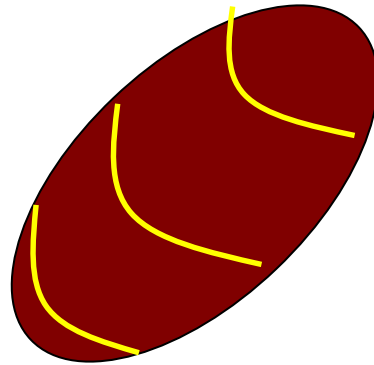
- volume
- shape



electric
point
charge



- volume
- shape
- magnetic moment



How to measure hyperfine interactions ?



- NMR
- NQR
- Mössbauer spectroscopy
- TDPAC
- Laser spectroscopy
- LTNO
- NMR/ON
- PAD
- ...

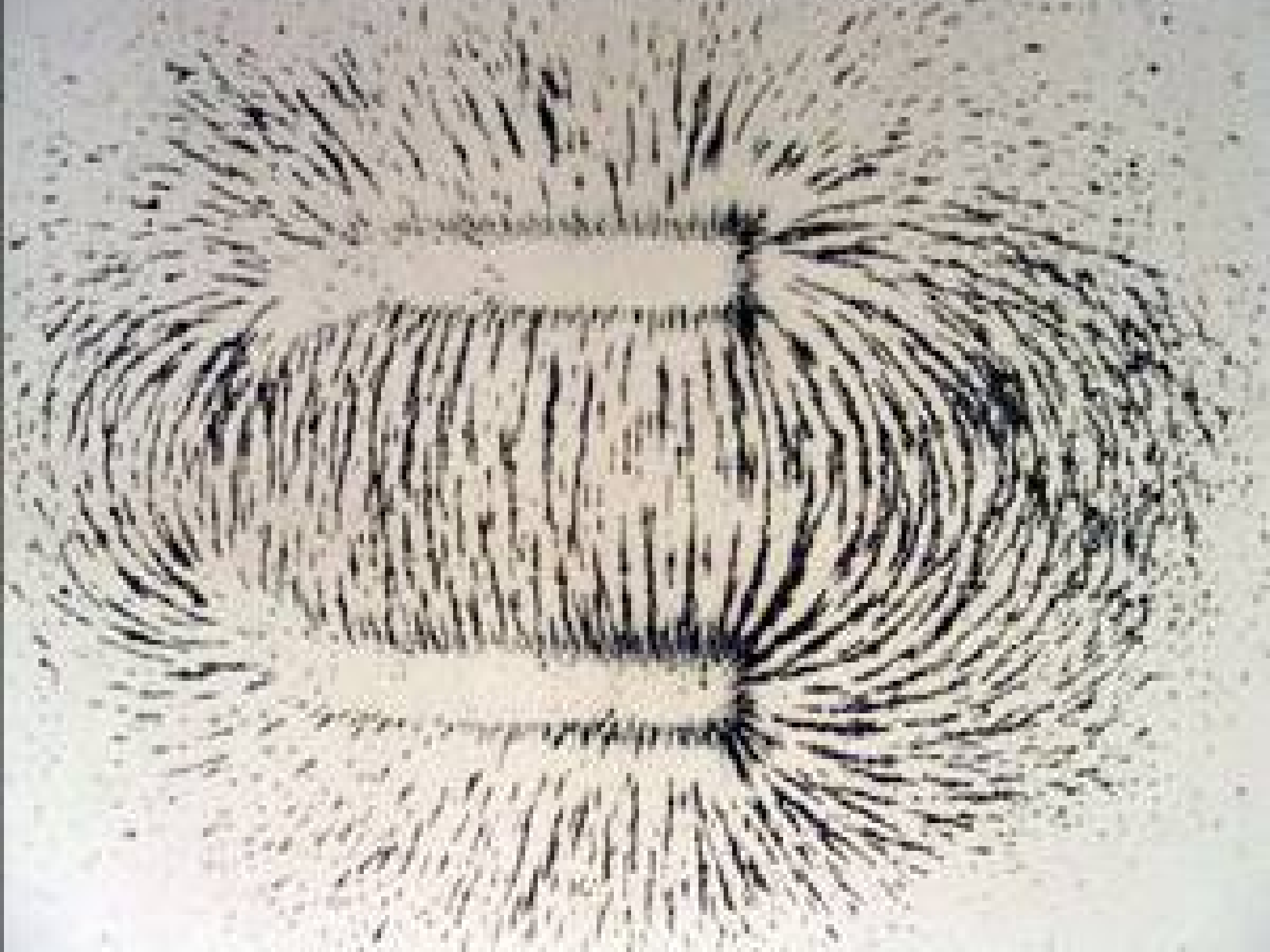
This talk:

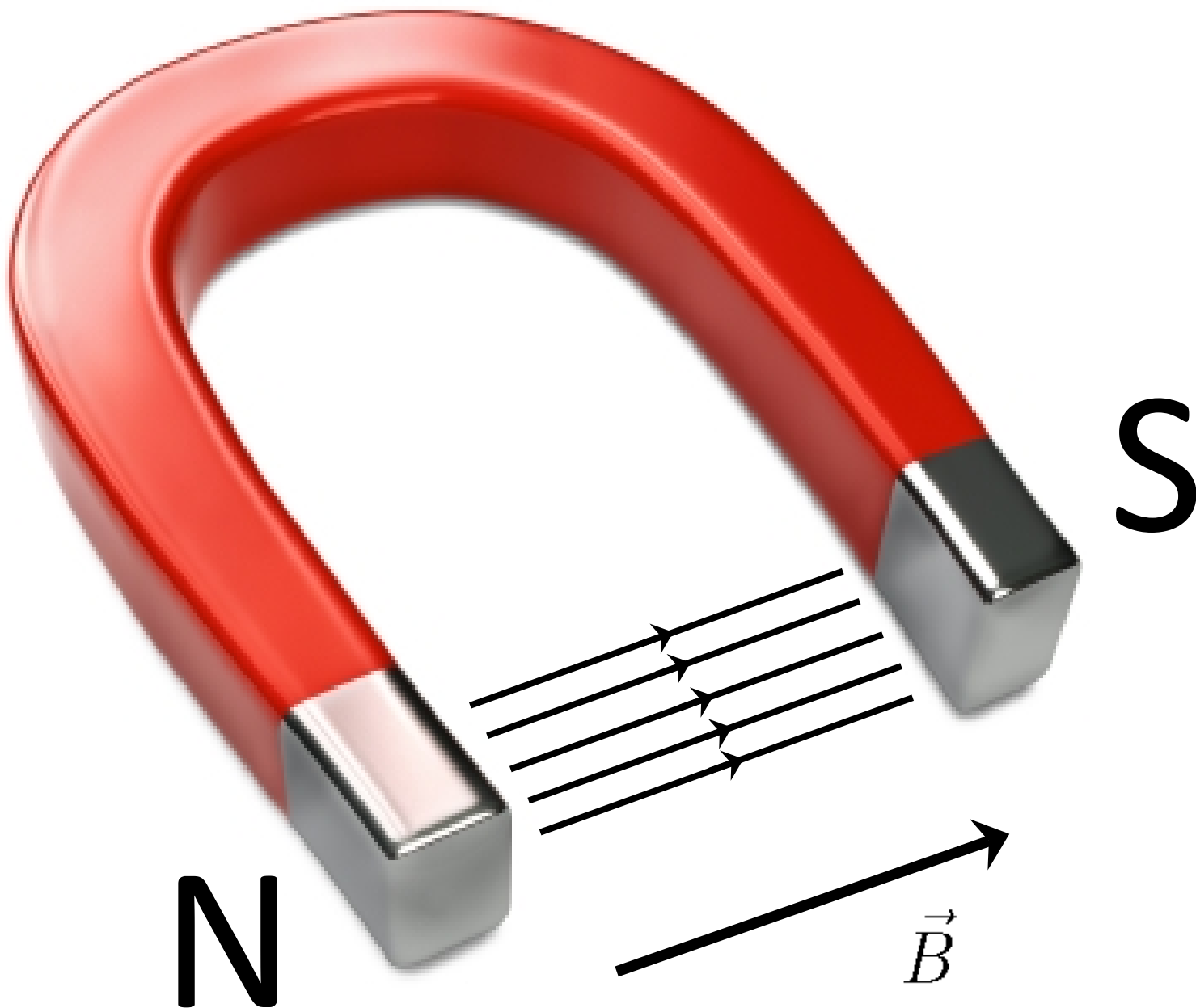
- Hyperfine physics
- How to calculate HFF with WIEN2k

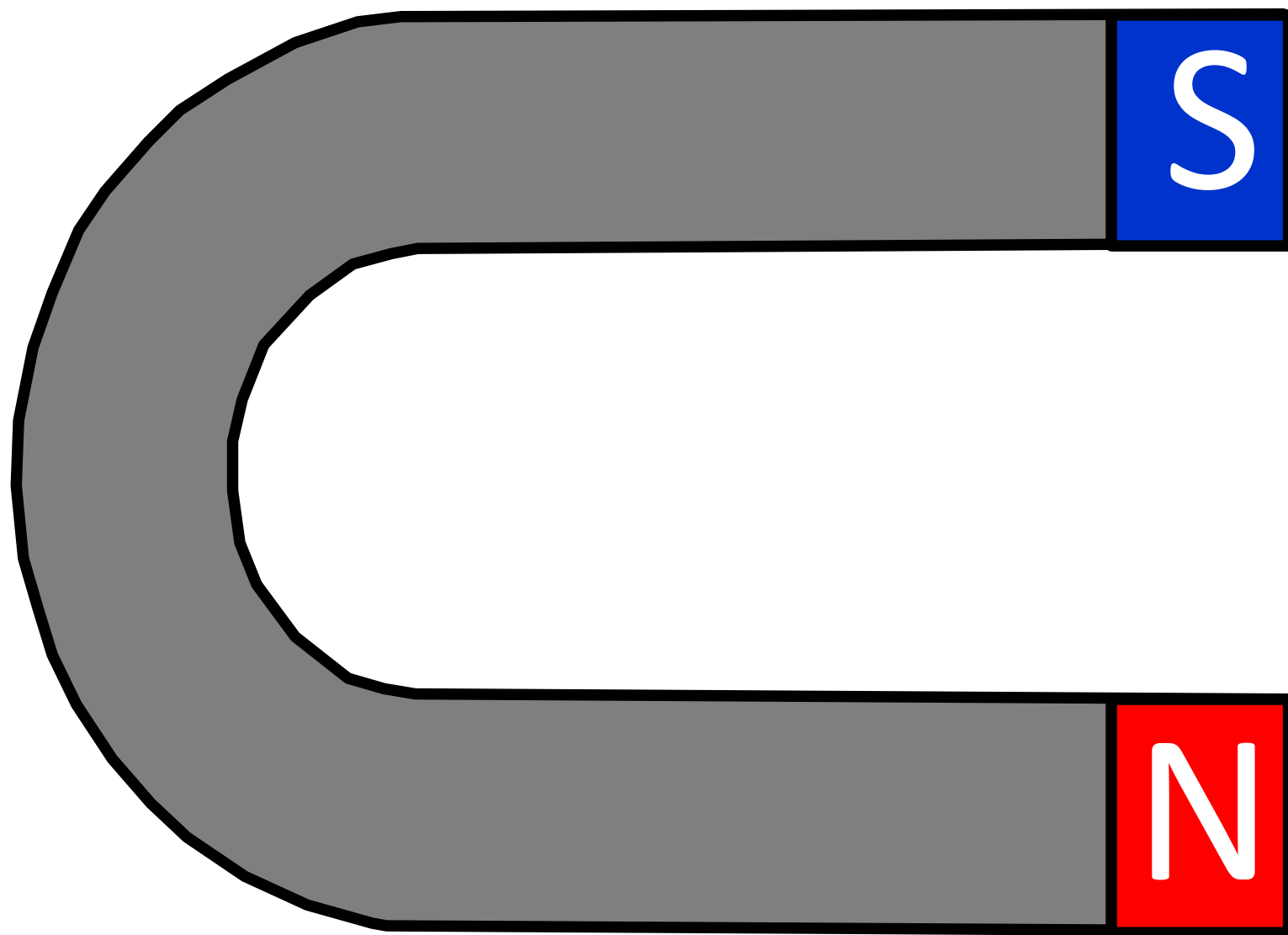
Content

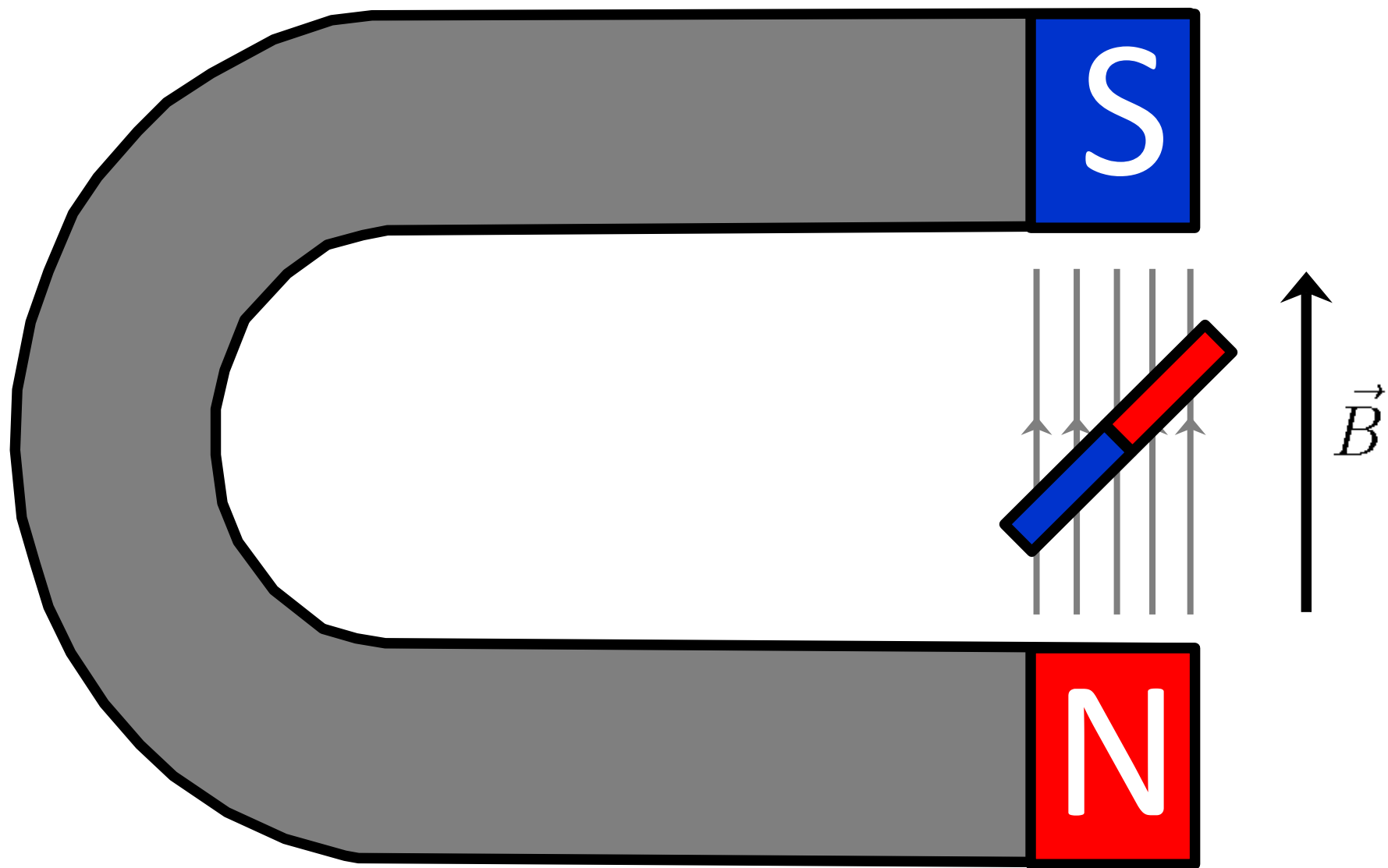
- Definitions
- magnetic hyperfine interaction
- electric quadrupole interaction
- isomer shift
- summary

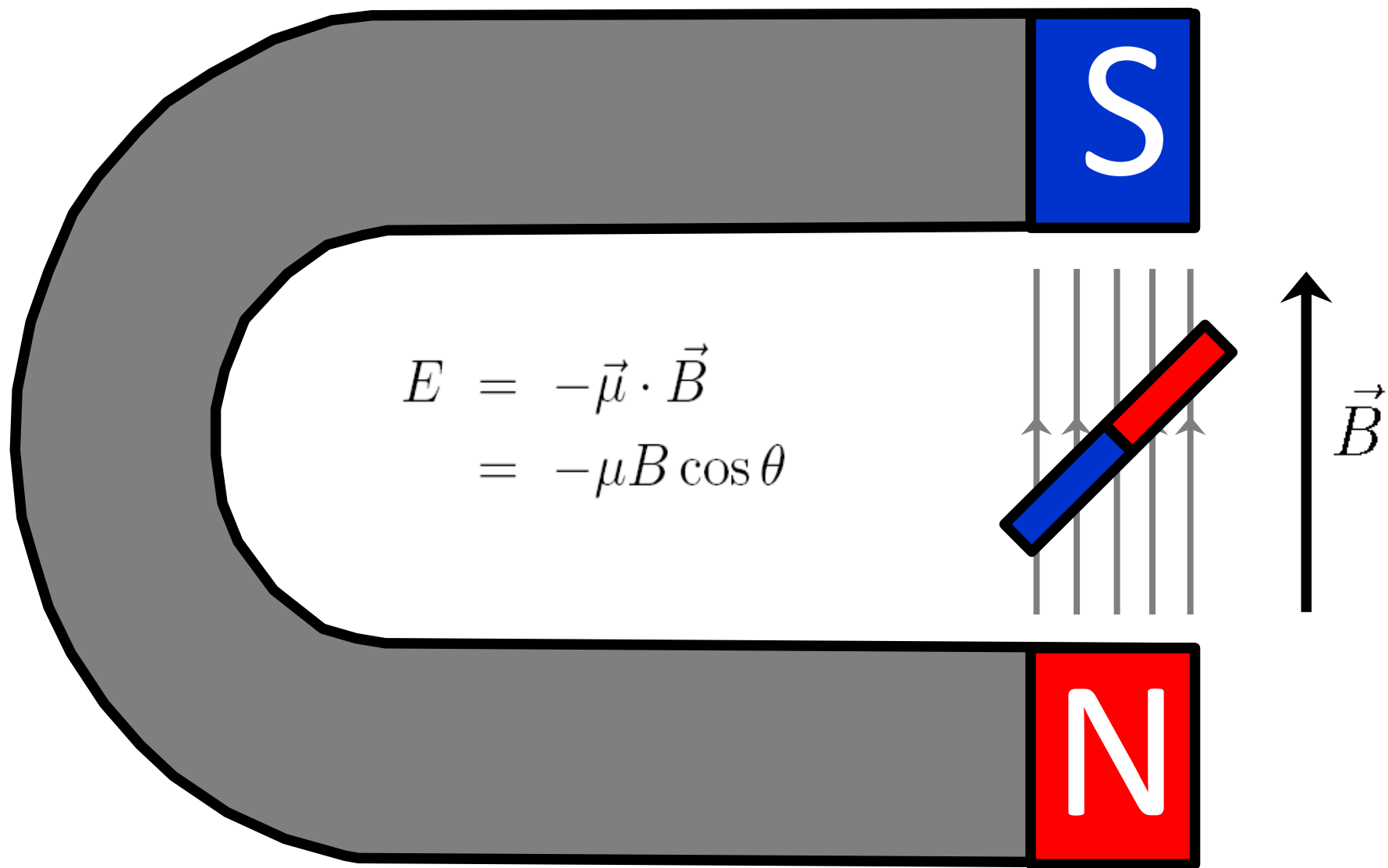


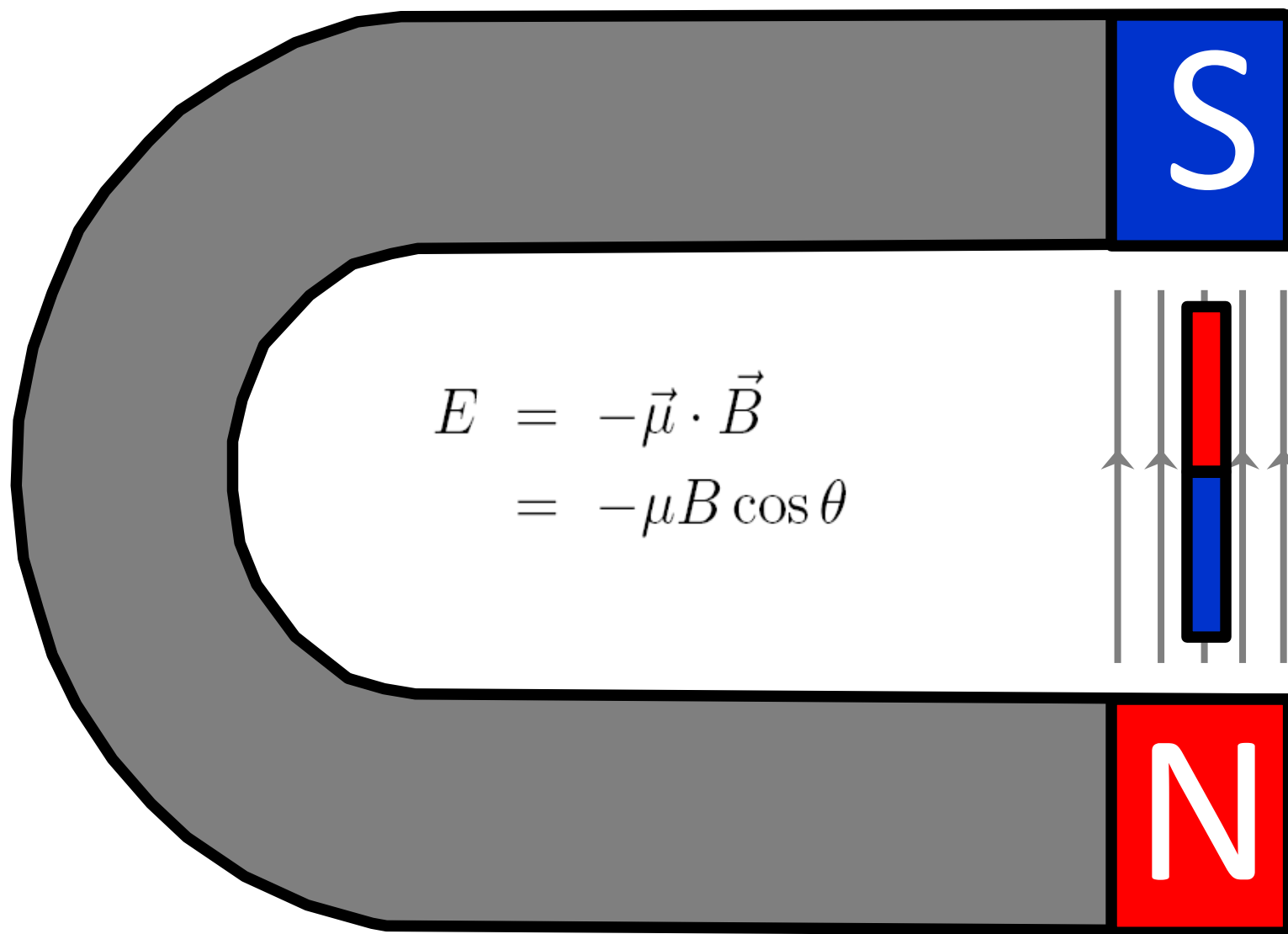


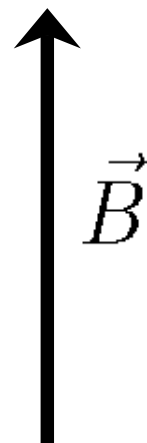
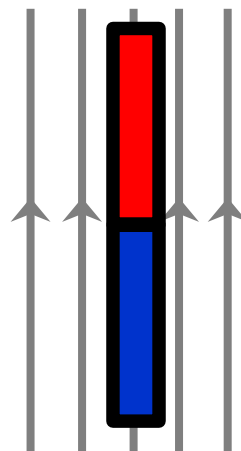
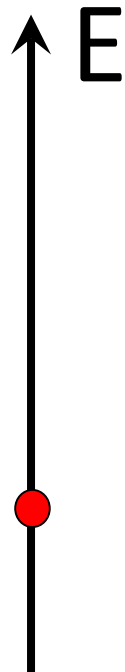
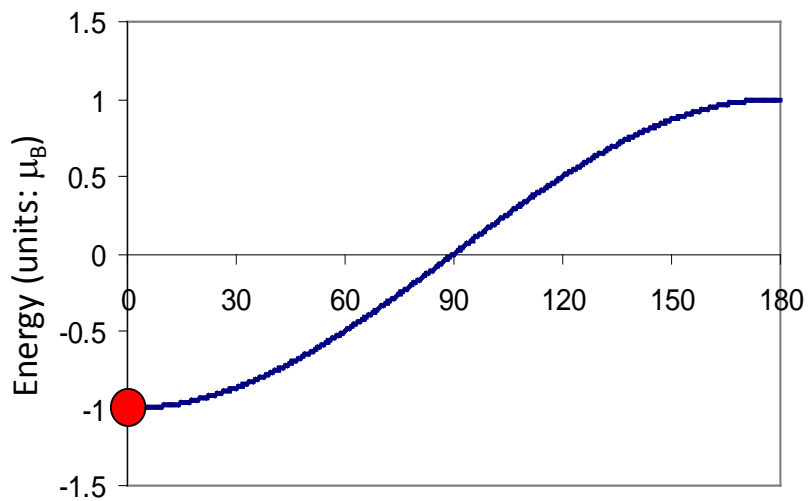




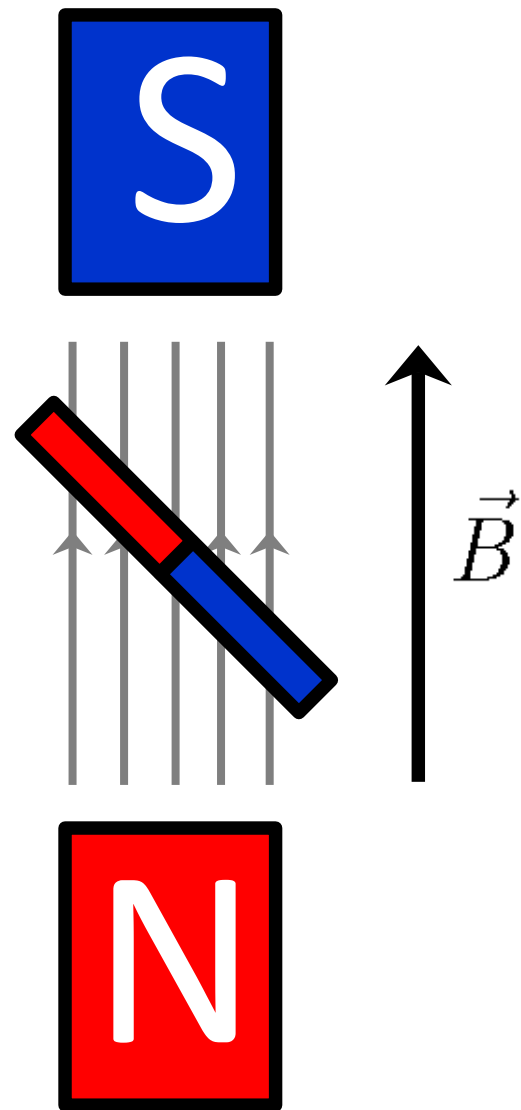
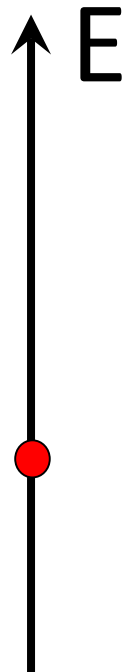
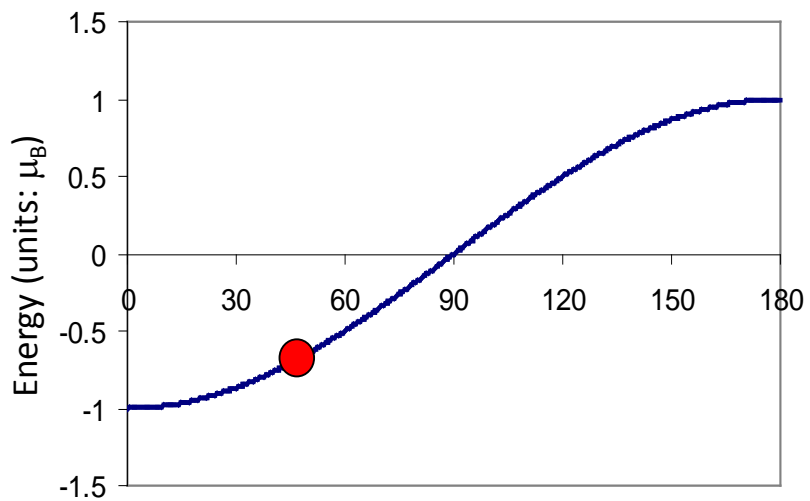




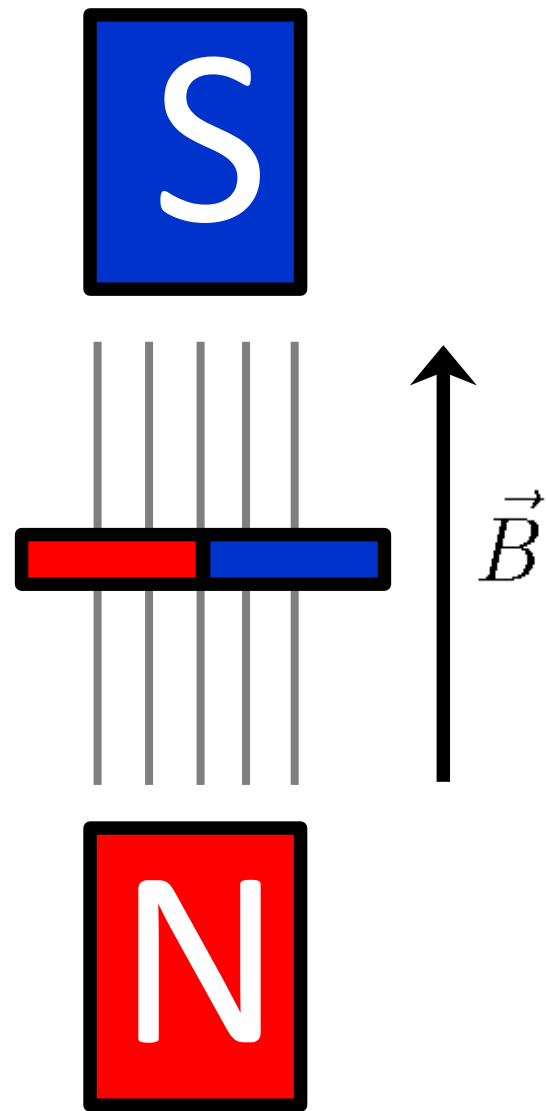
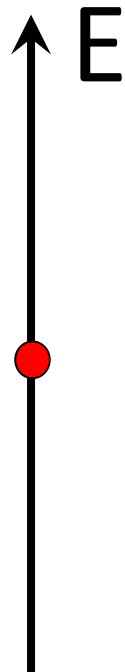
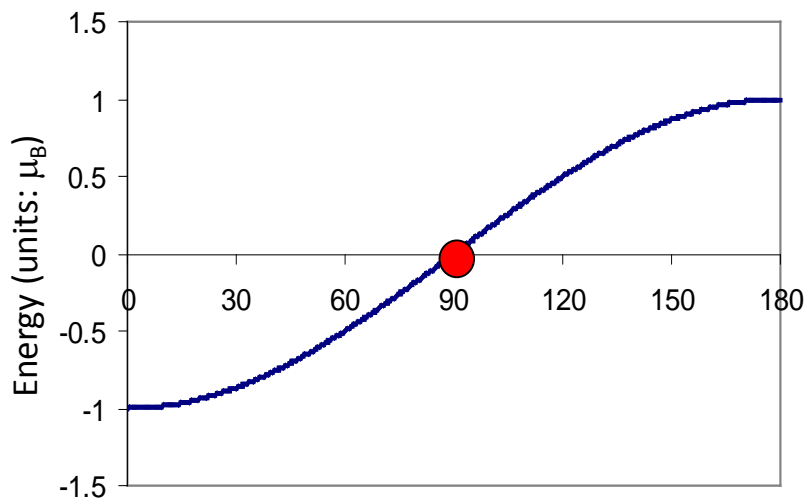




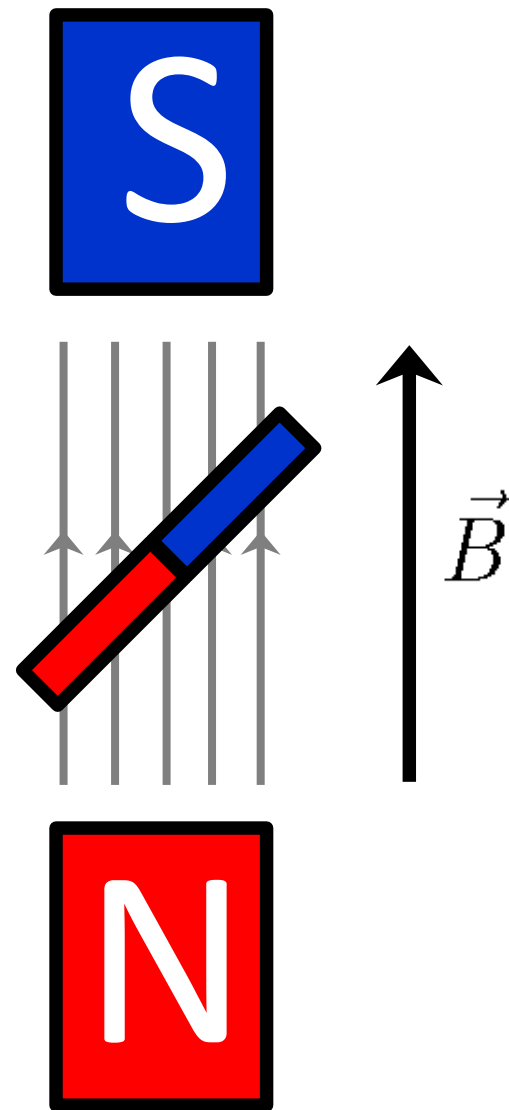
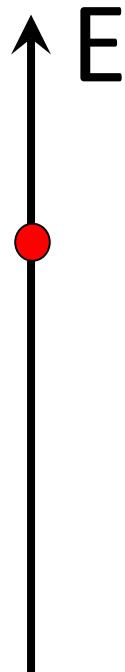
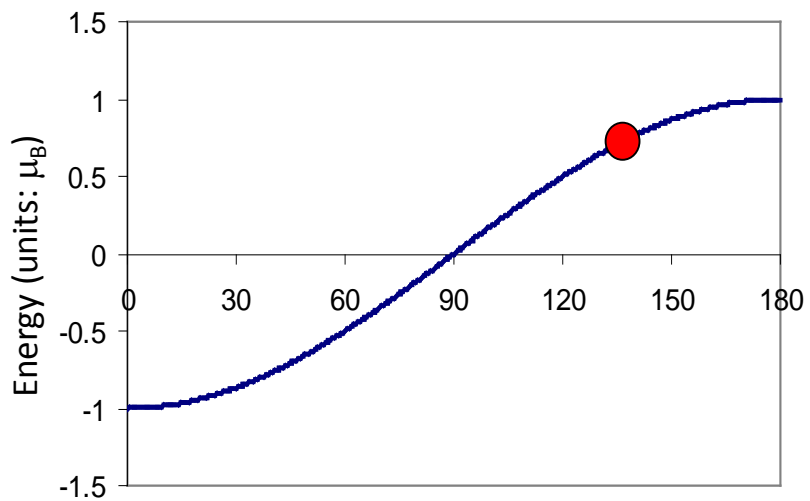
$$\begin{aligned}
 E &= -\vec{\mu} \cdot \vec{B} \\
 &= -\mu B \cos \theta
 \end{aligned}$$



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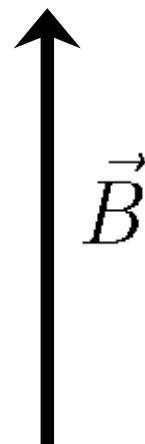
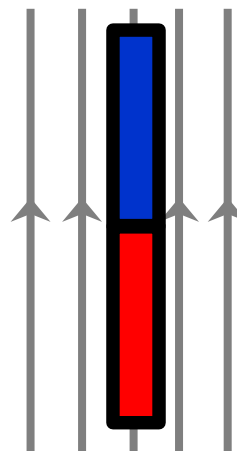
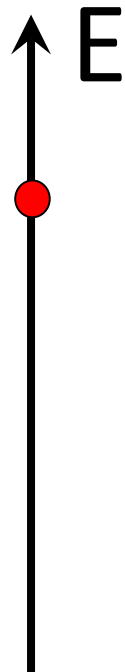
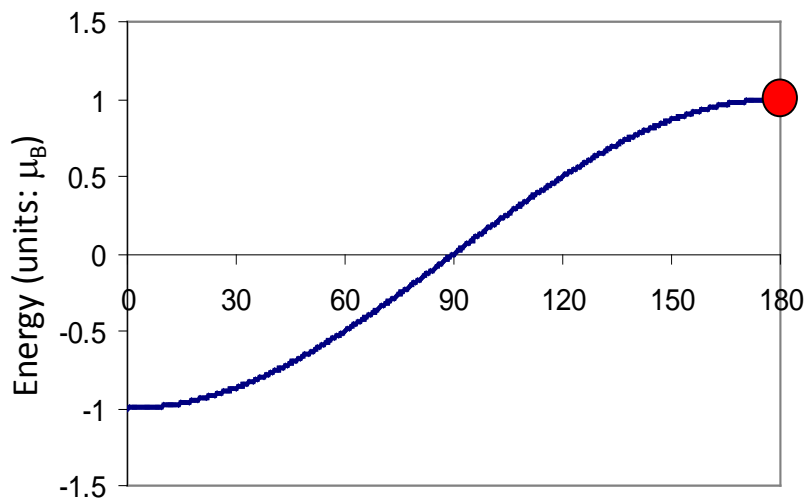


$$\begin{aligned}
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 &= -\mu B \cos \theta
 \end{aligned}$$

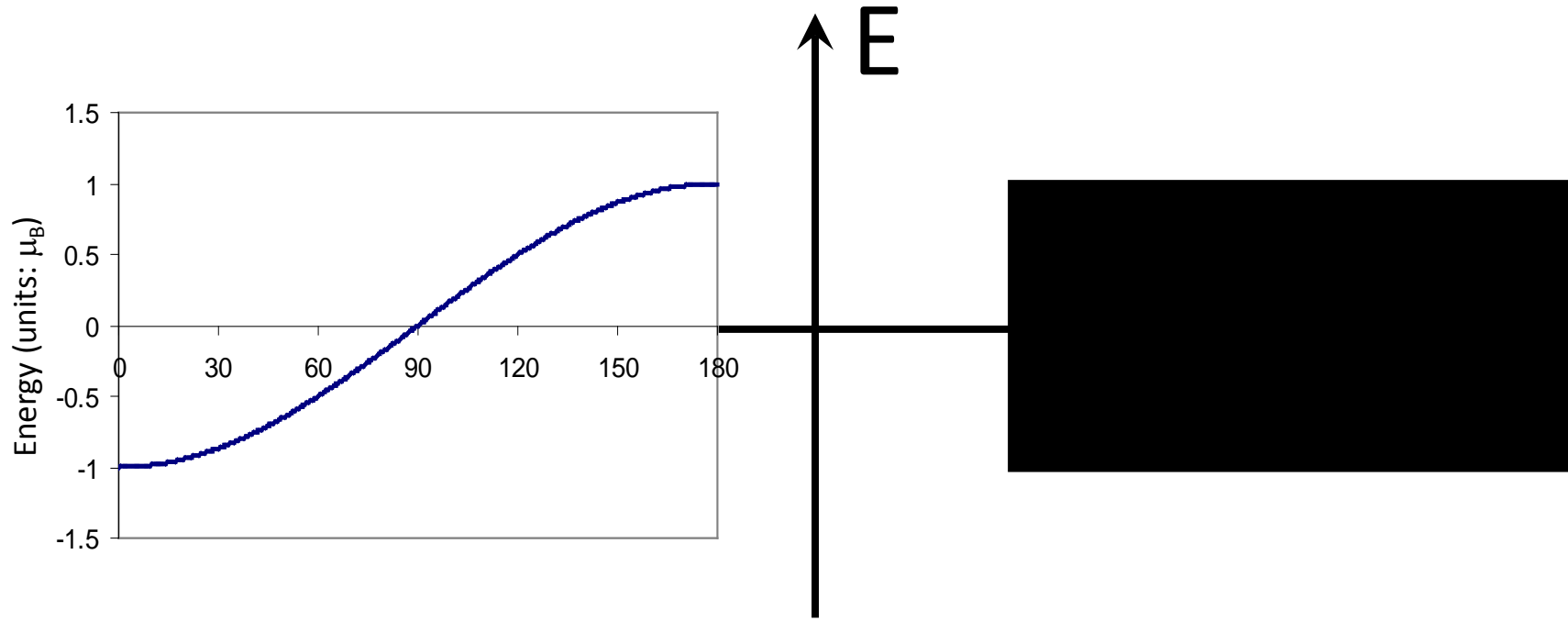


$$E = -\vec{\mu} \cdot \vec{B}$$

$$= -\mu B \cos \theta$$

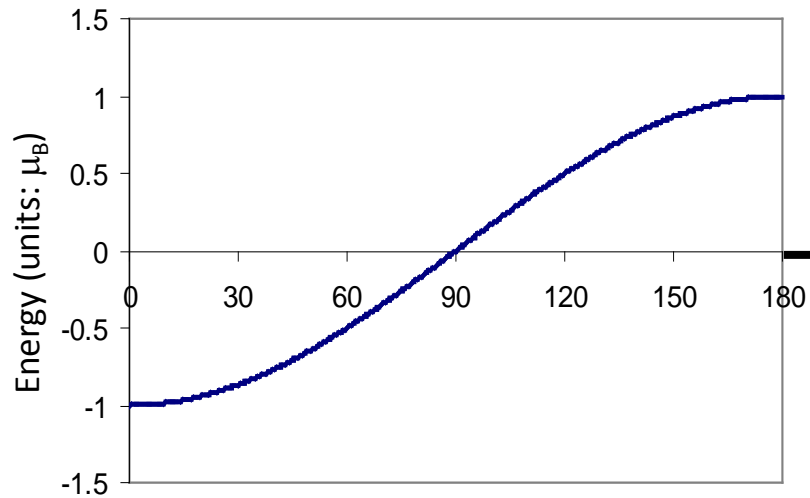


$$\begin{aligned}
 E &= -\vec{\mu} \cdot \vec{B} \\
 &= -\mu B \cos \theta
 \end{aligned}$$

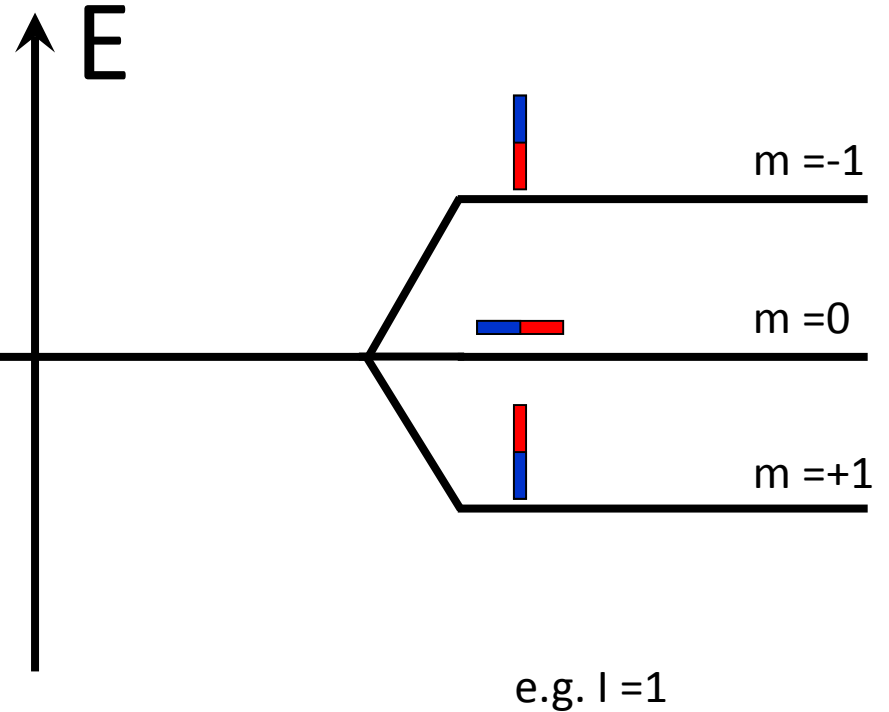


$$\begin{aligned} E &= -\vec{\mu} \cdot \vec{B} \\ &= -\mu B \cos \theta \end{aligned}$$

Classical



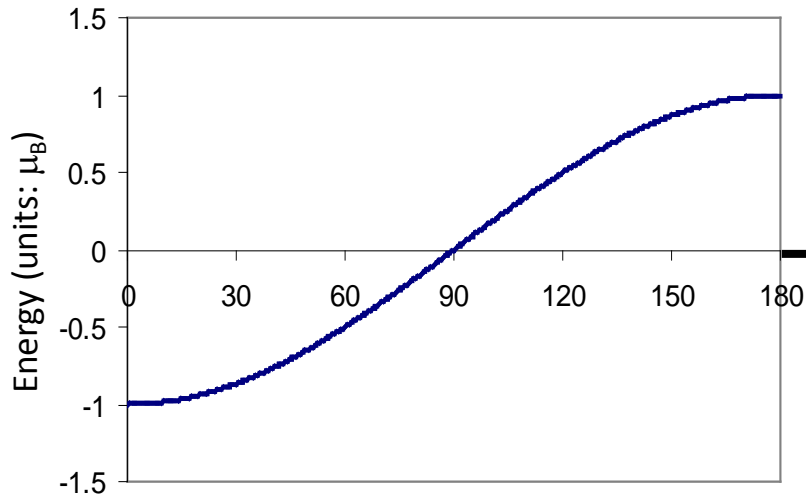
Quantum
(=quantization)



$$\begin{aligned} E &= -\vec{\mu} \cdot \vec{B} \\ &= -\mu B \cos \theta \end{aligned}$$

$$\hat{\mu}_I = \frac{\mu}{I \hbar} \hat{\mathbf{I}}$$

Classical



Quantum
(=quantization)

E

A diagram showing the quantization of energy levels. A vertical axis labeled 'E' has a horizontal line at zero. To the right, three horizontal lines represent energy levels labeled $m = -1$, $m = 0$, and $m = +1$. The $m = 0$ level is at zero energy. The $m = -1$ level is at a positive energy, and the $m = +1$ level is at a negative energy. Small vertical bars with blue and red segments are placed on each level to represent magnetic dipoles.

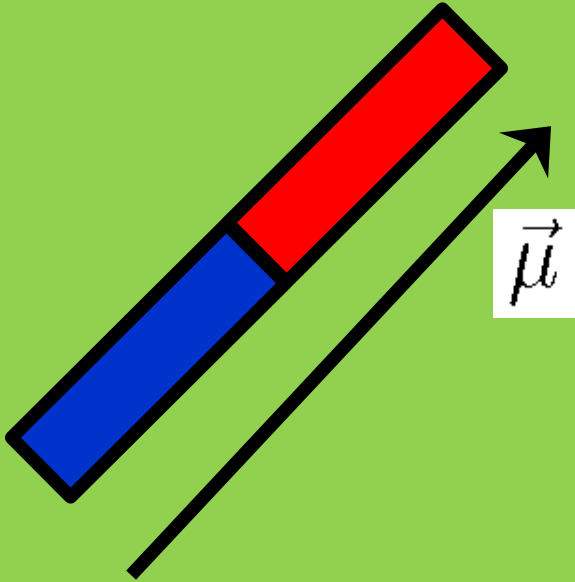
e.g. $l = 1$

Hamiltonian :

$$\hat{H} = -\frac{\mu B}{I \hbar} \hat{I}_z$$

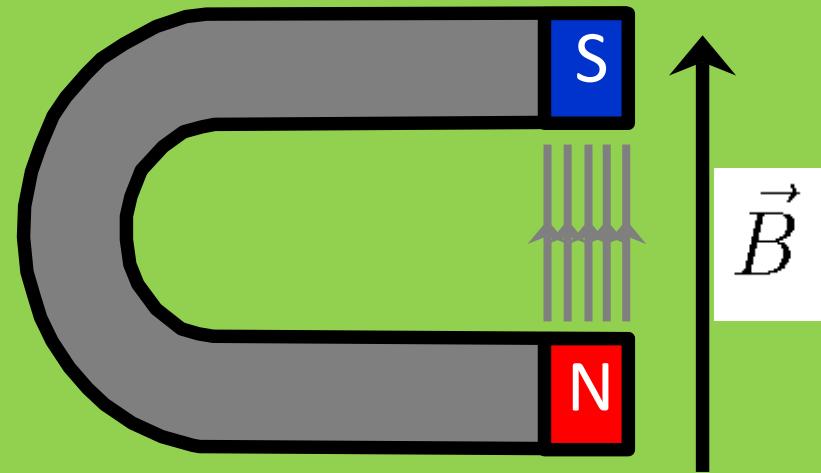
nuclear property

(vector)



electron property

(vector)



interaction energy (dot product) :

$$E = -\vec{\mu} \cdot \vec{B}$$

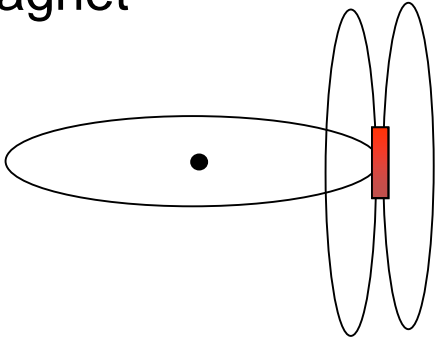
Source of magnetic fields at a nuclear site in an atom/solid

$$B_{\text{tot}} = B_{\text{dip}} + B_{\text{orb}} + B_{\text{fermi}} + B_{\text{lat}}$$

Source of magnetic fields at a nuclear site in an atom/solid

$$B_{\text{tot}} = B_{\text{dip}} + B_{\text{orb}} + B_{\text{fermi}} + B_{\text{lat}}$$

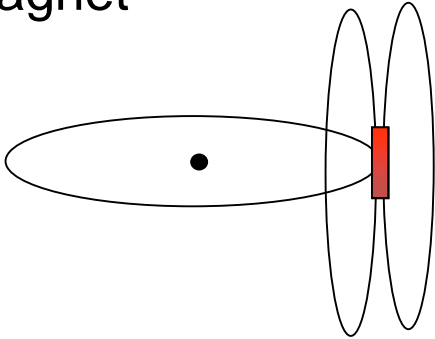
➤ B_{dip} = electron as bar magnet



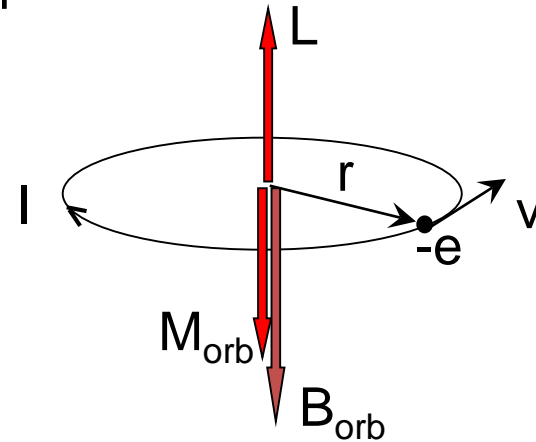
Source of magnetic fields at a nuclear site in an atom/solid

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➤ B_{dip} = electron as bar magnet



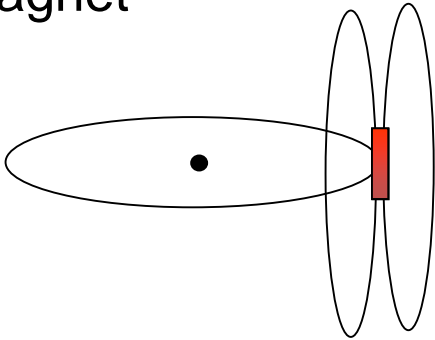
➤ B_{orb} = electron as current loop



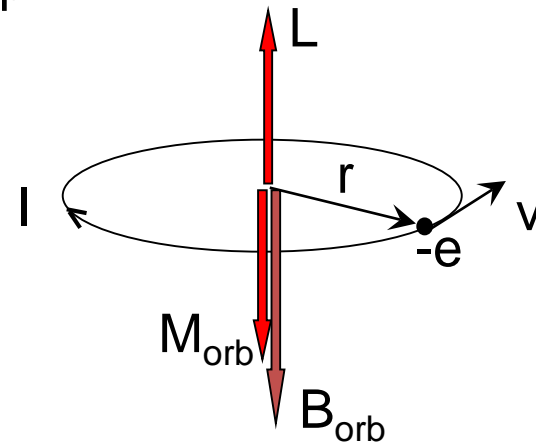
Source of magnetic fields at the nuclear site in an atom/solid

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{dip}} + \mathbf{B}_{\text{orb}} + \mathbf{B}_{\text{fermi}} + \mathbf{B}_{\text{lat}}$$

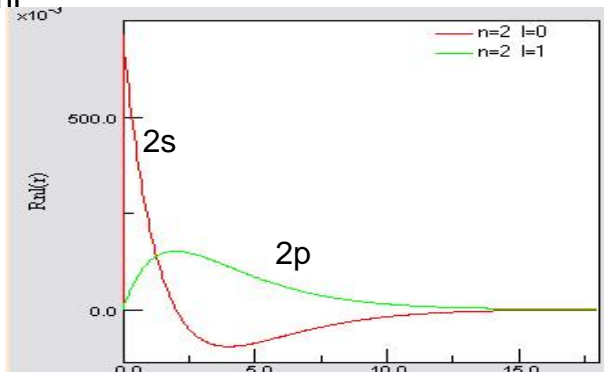
➤ \mathbf{B}_{dip} = electron as bar magnet



➤ \mathbf{B}_{orb} = electron as current loop



➤ $\mathbf{B}_{\text{Fermi}}$ = electron at nucleus

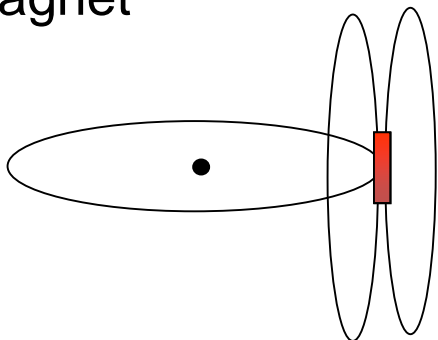


$$-\frac{2\mu_B\mu_0}{3} \left(|\psi_{e,\uparrow}(\mathbf{0})|^2 - |\psi_{e,\downarrow}(\mathbf{0})|^2 \right)$$

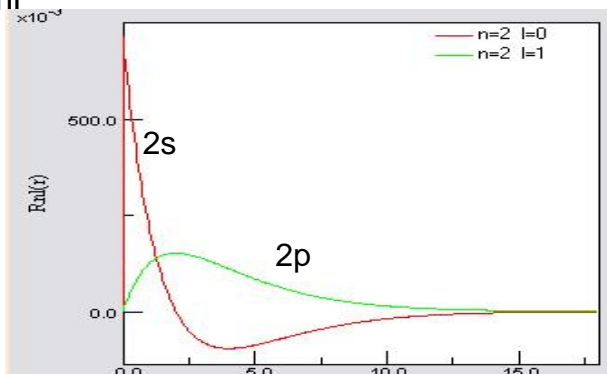
Source of magnetic fields at a nuclear site in an atom/solid

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{dip}} + \mathbf{B}_{\text{orb}} + \mathbf{B}_{\text{fermi}} + \mathbf{B}_{\text{lat}}$$

➤ \mathbf{B}_{dip} = electron as bar magnet

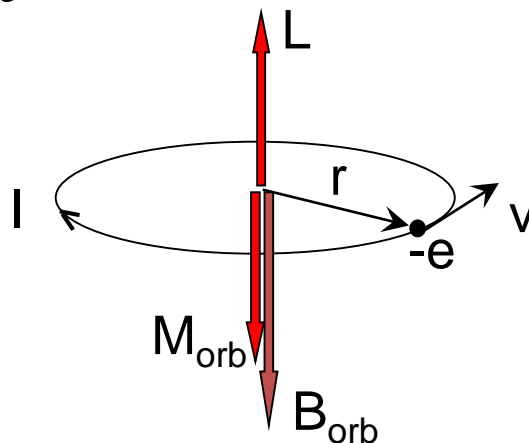


➤ $\mathbf{B}_{\text{Fermi}}$ = electron at nucleus

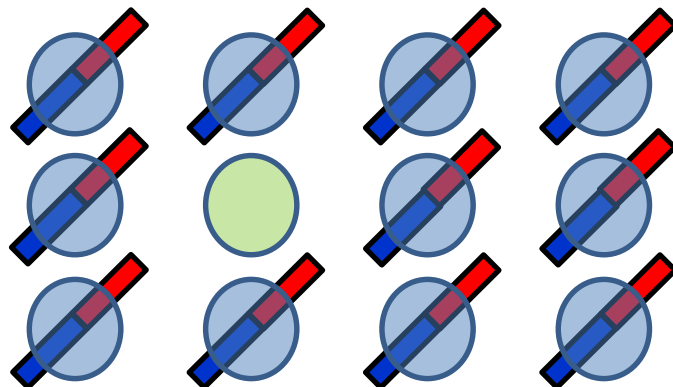


$$-\frac{2\mu_B\mu_0}{3} \left(|\psi_{e,\uparrow}(\mathbf{0})|^2 - |\psi_{e,\downarrow}(\mathbf{0})|^2 \right)$$

➤ \mathbf{B}_{orb} = electron as current loop



➤ \mathbf{B}_{lat} = neighbours as bar magnets



How to do it in WIEN2k ?

Magnetic hyperfine field

In regular scf file:

:HFFxxx (Fermi contact contribution)

After post-processing with LAPWDM :

- orbital hyperfine field ("3 3" in case.indmc)
- dipolar hyperfine field ("3 5" in case.indmc)

in case.scfdmup

```
----- top of file: case.indm -----  
-9.          Emin cutoff energy  
1           number of atoms for which density matrix is calculated  
1 1 2      index of 1st atom, number of L's, L1  
0 0        r-index, (l,s)-index  
----- bottom of file -----
```

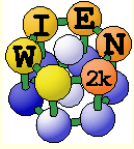
After post-processing with DIPAN :

- lattice contribution
- in case.outputdipan

more info:

UG 7.8 (lapwdm)

UG 8.3 (dipan)



Mössbauer spectroscopy:

- Isomer shift: $\delta = \alpha (\rho_0^{\text{Sample}} - \rho_0^{\text{Reference}})$; $\alpha = -.291 \text{ au}^3\text{mm s}^{-1}$
 - *proportional to the electron density ρ at the nucleus*

- Magnetic Hyperfine fields: $B_{\text{tot}} = B_{\text{contact}} + B_{\text{orb}} + B_{\text{dip}}$

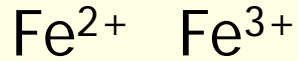
- $B_{\text{contact}} = 8\pi/3 \mu_B [\rho_{up}(0) - \rho_{dn}(0)]$... *spin-density at the nucleus*

$$\vec{B}_{\text{orb}} = 2\mu_B \langle \Phi | \frac{S(r)}{r^3} \vec{l} | \Phi \rangle$$
 ... *orbital-moment*

$$\vec{B}_{\text{dip}} = 2\mu_B \langle \Phi | \frac{S(r)}{r^3} [3(\vec{s} \hat{r}) \hat{r} - \vec{s}] | \Phi \rangle$$
 ... *spin-moment*

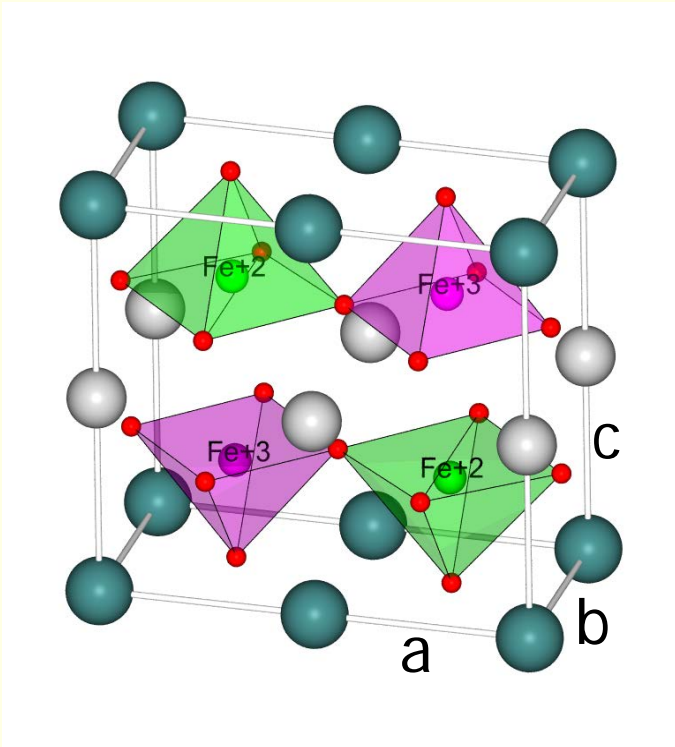
$S(r)$ is reciprocal of the relativistic mass enhancement $S(r) = \left[1 + \frac{\epsilon - V(r)}{2mc^2} \right]^{-1}$

Charged ordered



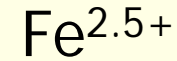
CO structure: $Pmma$

$a:b:c=2.09:1:1.96$ (20K)



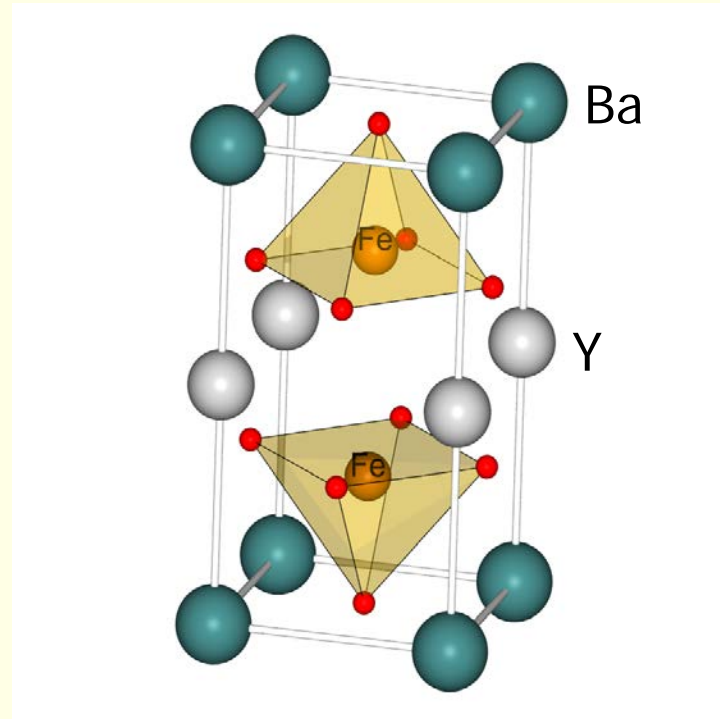
■ Fe^{2+} and Fe^{3+} chains along b

Valence mixed



VM structure: $Pmmm$

$a:b:c=1.003:1:1.93$ (340K)



$\text{Fe}^{2.5+}$

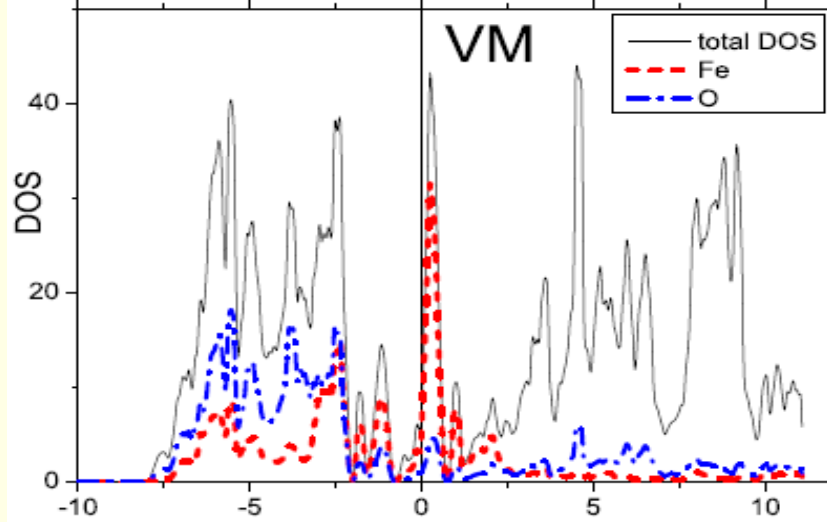
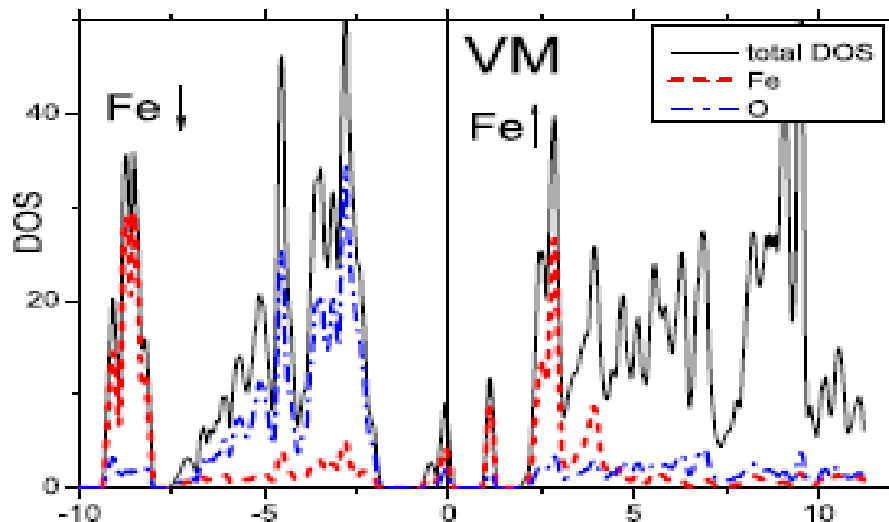
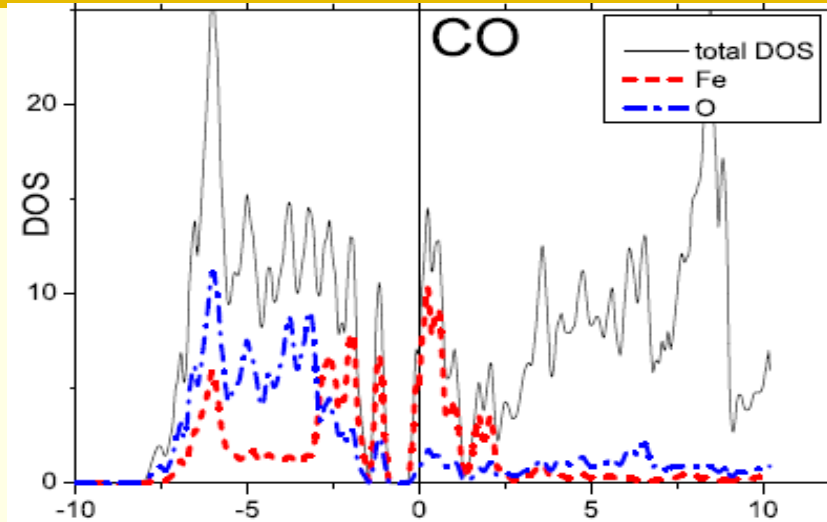
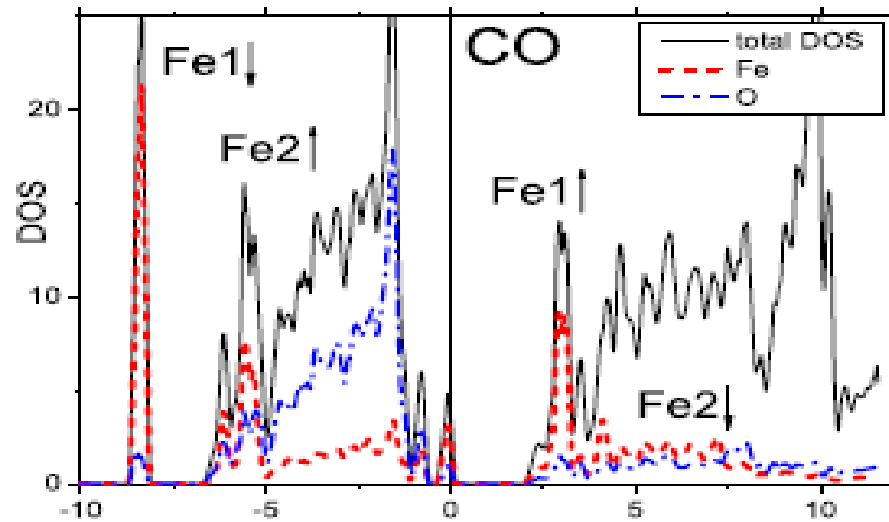


DOS: GGA+U vs. GGA

GGA+U

GGA

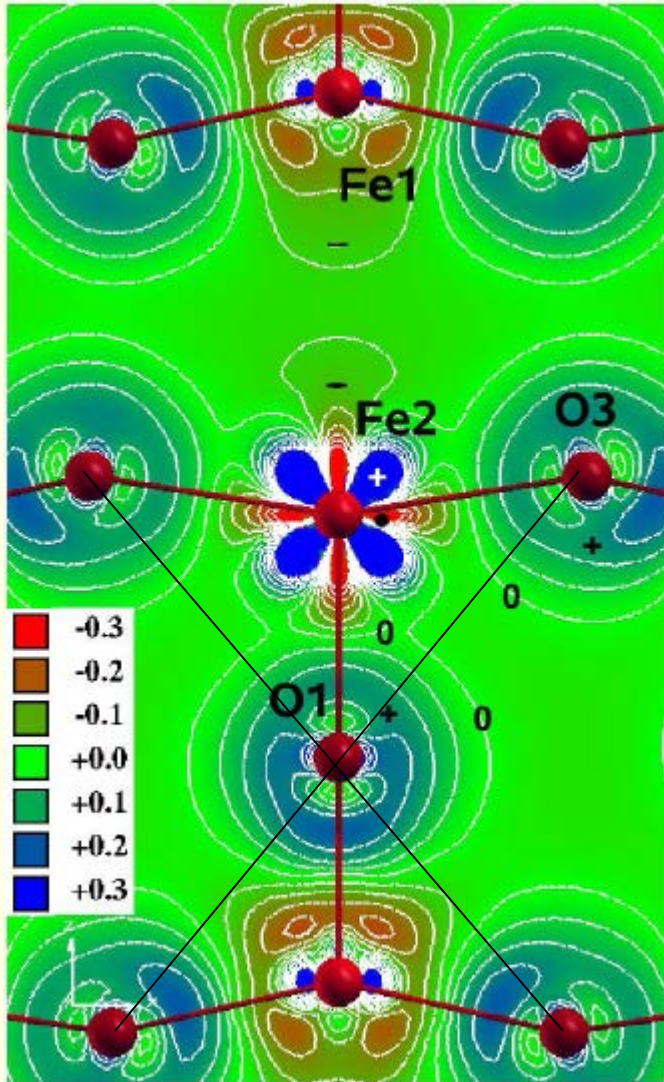
single lower Hubbard-band in VM splits in CO with Fe^{3+} states lower than Fe^{2+}



Energy [eV]

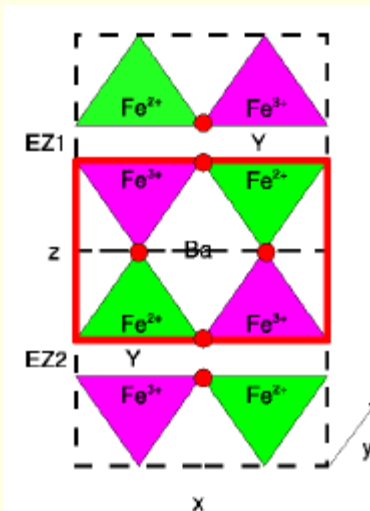
Energy [eV]

CO phase

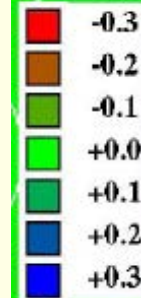
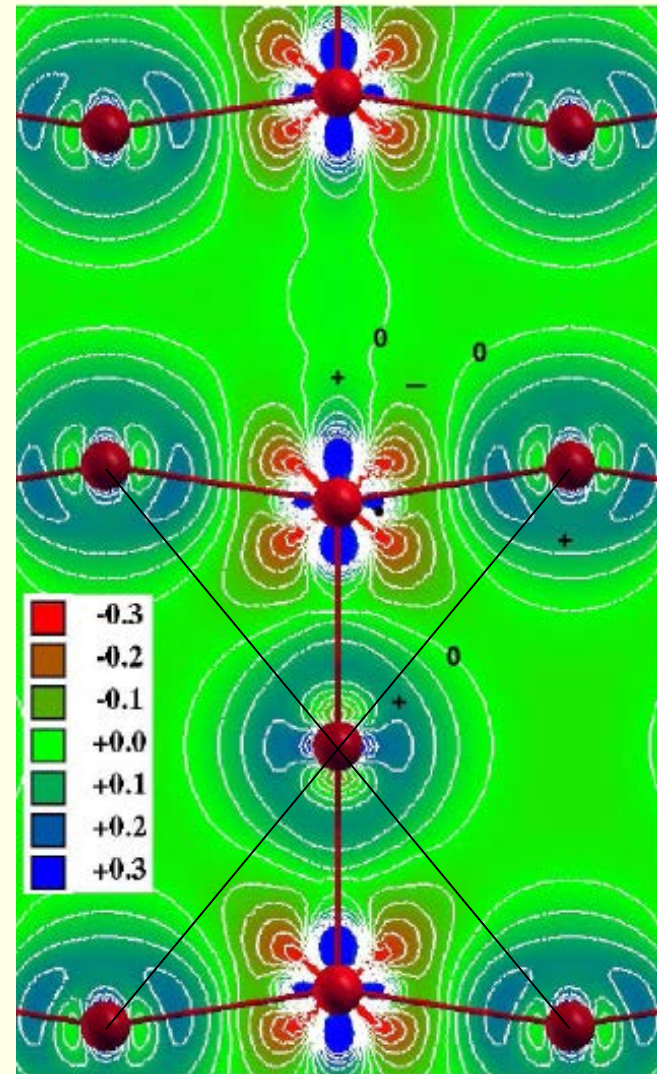


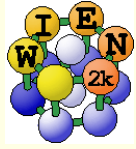
Fe^{2+} : d-xz
 Fe^{3+} : d-x²
 O1 and O3: polarized toward Fe^{3+}

Fe: d-z² Fe-Fe interaction
 O: symmetric



VM phase





YBaFe₂O₅ HFF, IS and EFG with GGA+U, LDA/GGA



TABLE VIII: Hyperfine fields B (in Tesla), isomer shifts δ (mm/s) and quadrupole coupling constants eQV_{zz} (mm/s) for the CO phase for various exchange and correlation potentials and experiment⁸⁻¹⁰.

| CO | | exp. | GGA+U | | LDA | GGA |
|--|----------------|----------------------|--------|--------|-------|--------|
| | U_{eff} [eV] | — | 5 | 6 | — | — |
| Fe ²⁺ | B_{dip} | — | -16.29 | -16.49 | -6.68 | -12.67 |
| | B_{orb} | — | -6.73 | -6.90 | -9.57 | -6.34 |
| | $B_{contact}$ | — | 32.25 | 32.23 | 32.21 | 31.58 |
| | B_{tot} | ~ 8 | 9.23 | 8.83 | 15.96 | 12.57 |
| | δ | ~ 1 | 0.92 | 0.94 | 0.74 | 0.79 |
| | eQV_{zz} | 3.6 – 4 ^a | 3.66 | 3.74 | -0.82 | 2.60 |
| Fe ³⁺ | B_{dip} | — | -0.67 | -0.60 | 1.29 | 0.39 |
| | B_{orb} | — | -0.52 | -0.45 | -7.96 | -2.65 |
| | $B_{contact}$ | — | 37.65 | 38.28 | 29.64 | 31.63 |
| | B_{tot} | ~ 50 | 36.46 | 37.24 | 22.97 | 29.37 |
| | δ | ~ 0.4 | 0.33 | 0.30 | 0.50 | 0.47 |
| | eQV_{zz} | 1 – 1.5 ^a | 1.46 | 1.50 | 1.04 | -0.30 |
| ^a depending on rare earth ion | | | | | | |
| VM | | exp. | GGA+U | | LDA | GGA |
| | U_{eff} [eV] | — | 5 | 6 | — | — |
| Fe ^{2.5+} | B_{dip} | — | -3.00 | -2.98 | -2.13 | -2.83 |
| | B_{orb} | — | -3.11 | -2.99 | -5.47 | -4.56 |
| | $B_{contact}$ | — | 41.17 | 40.96 | 33.10 | 36.36 |
| | B_{tot} | ~ 30 | 35.06 | 34.98 | 25.50 | 28.98 |
| | δ | ~ 0.5 | 0.53 | 0.52 | 0.60 | 0.60 |
| | eQV_{zz} | ~ 0.1 | 0.12 | 0.13 | 0.19 | -0.27 |

HFF(Fe²⁺)

HFF(Fe³⁺)

HFF(Fe^{2.5+})

Content

- Definitions
- magnetic hyperfine interaction
- electric quadrupole interaction
- isomer shift
- summary



Electric Hyperfine-Interaction

- between nuclear charge distribution (σ) and external potential

$$E = \int \sigma_n(x) V(x) dx$$

- Taylor-expansion at the nuclear position

$$E = V_0 Z$$

direction independent constant

$$+ \sum_i \frac{\partial V(0)}{\partial x_i} \int \sigma(x) x_i dx$$

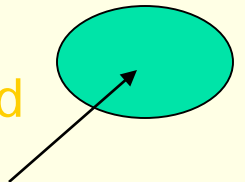
electric field \times
nuclear dipol moment (=0)

$$+ \frac{1}{2} \sum_{ij} \frac{\partial^2 V(0)}{\partial x_i \partial x_j} \iint \sigma(x) x_i x_j dx$$

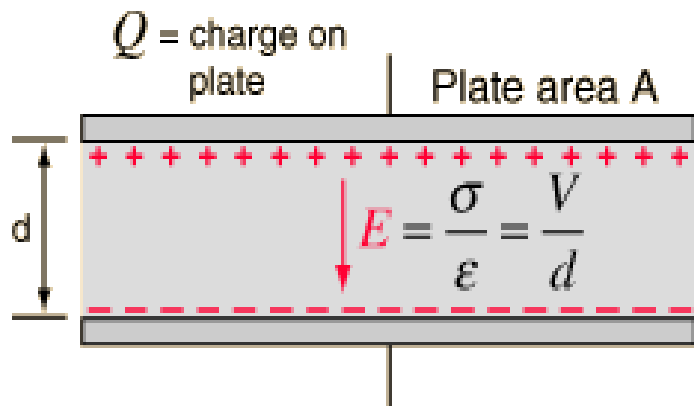
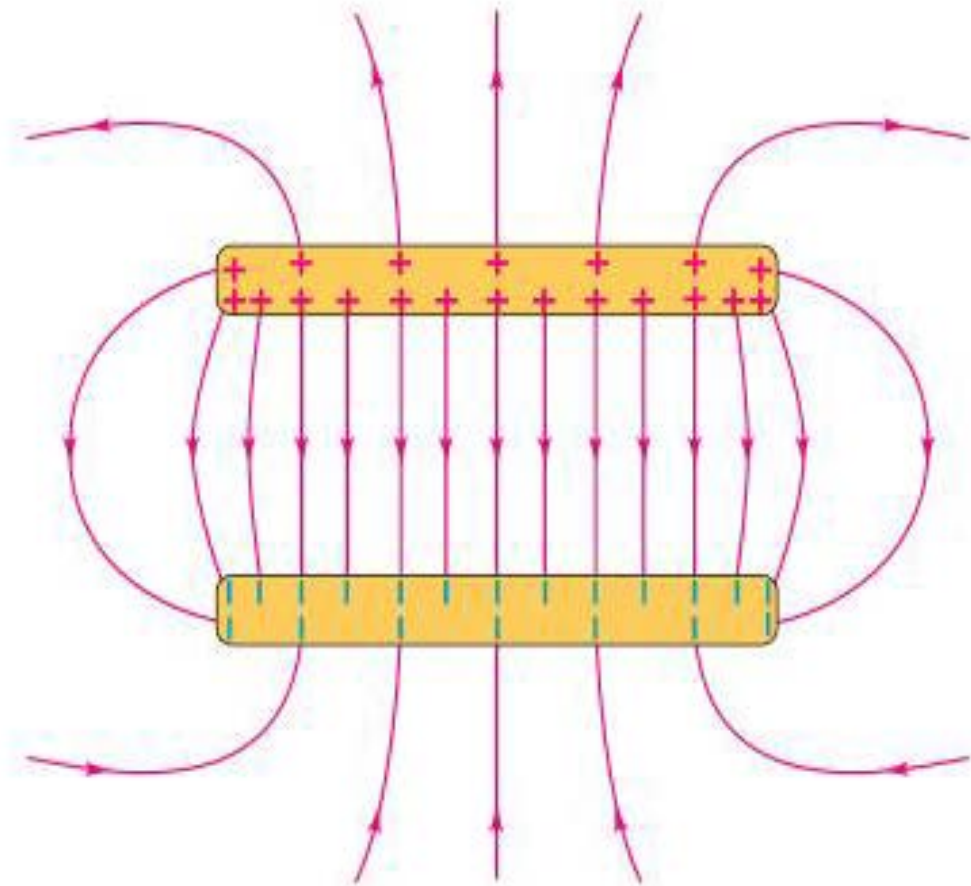
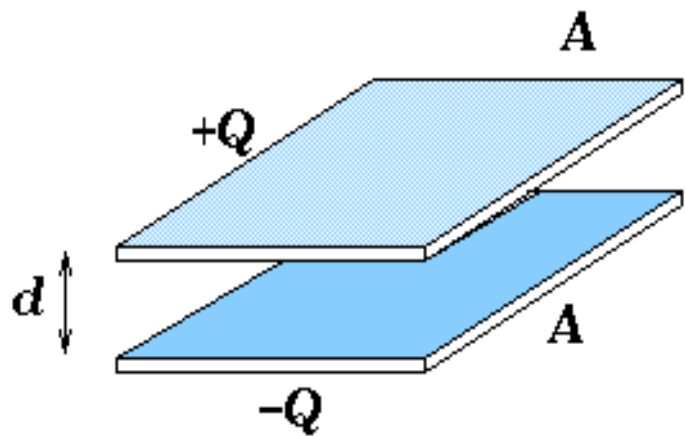
electric field gradient \times
nuclear quadrupol moment Q

+ ...

higher terms neglected

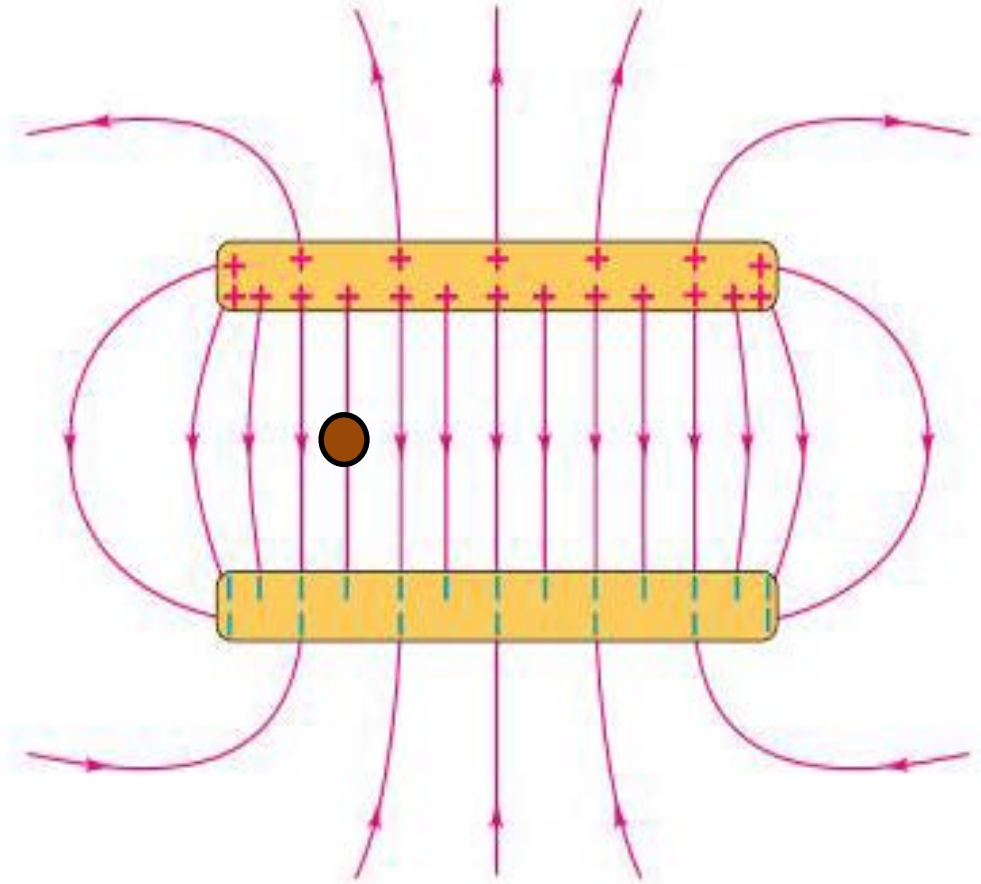


nucleus with charge Z , but not a sphere



- Force on a point charge:

$$\vec{F} = Q\vec{E}$$

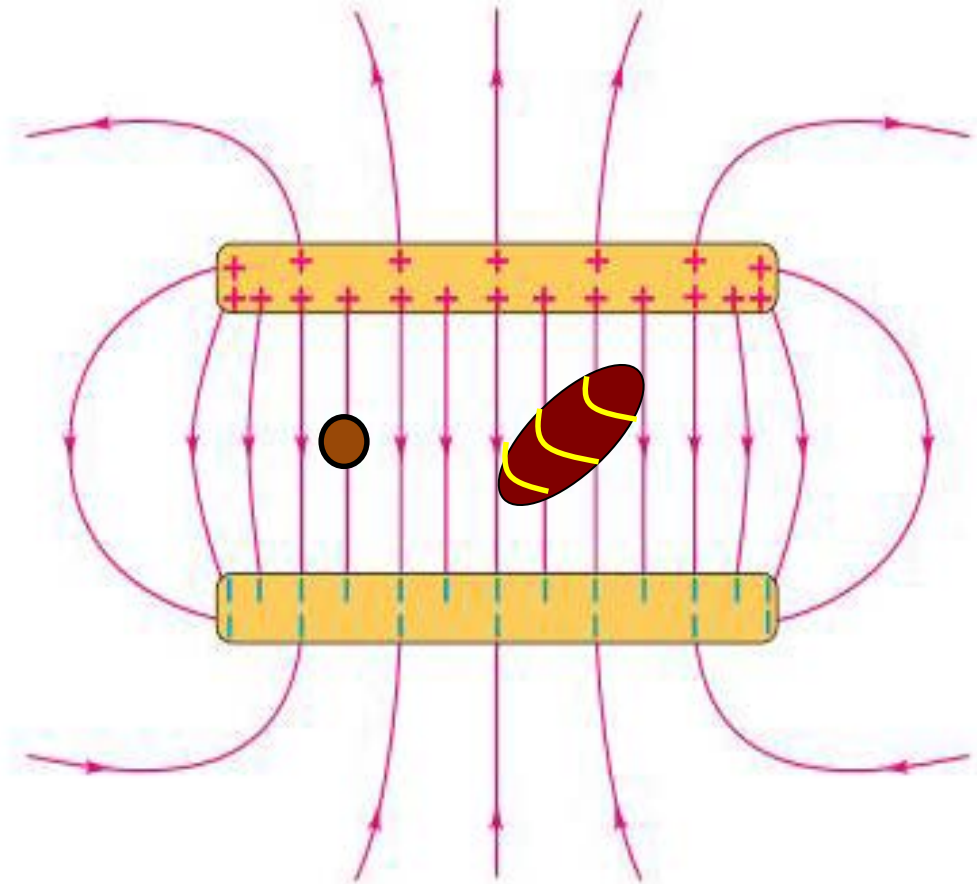


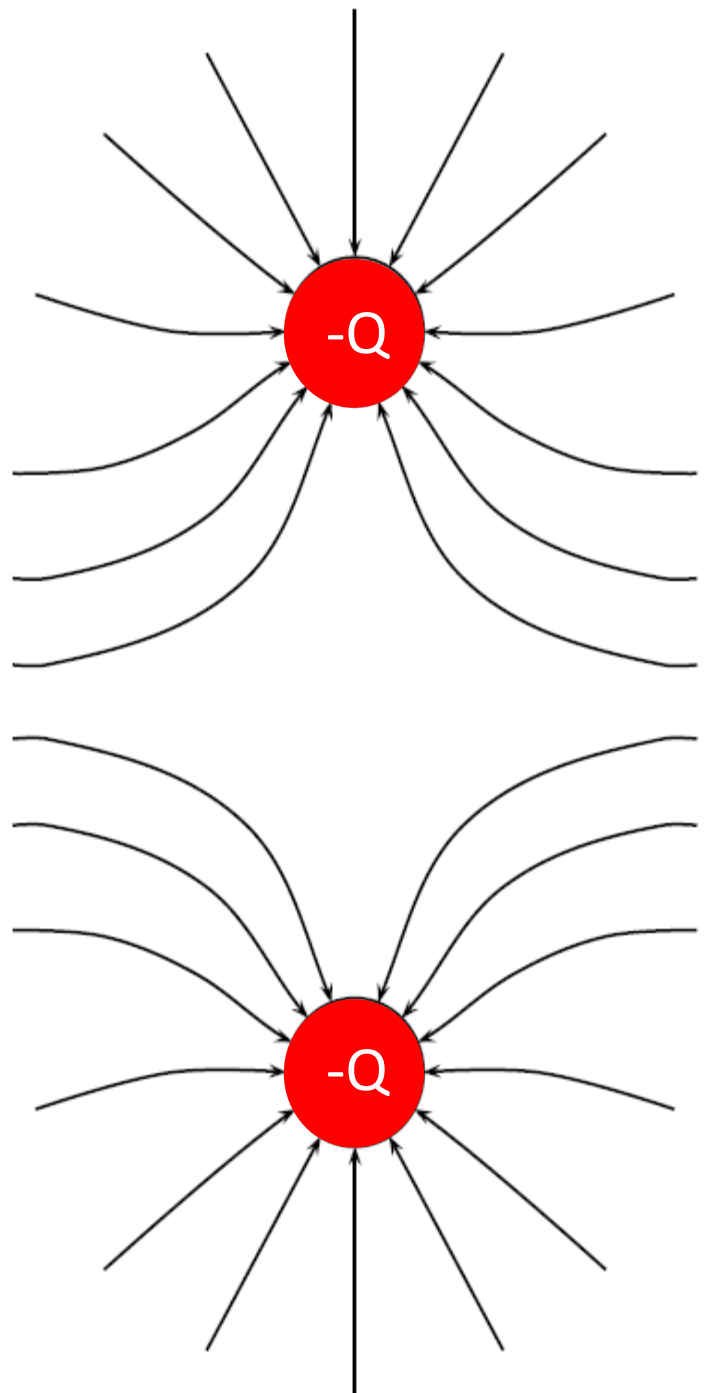
- Force on a point charge:

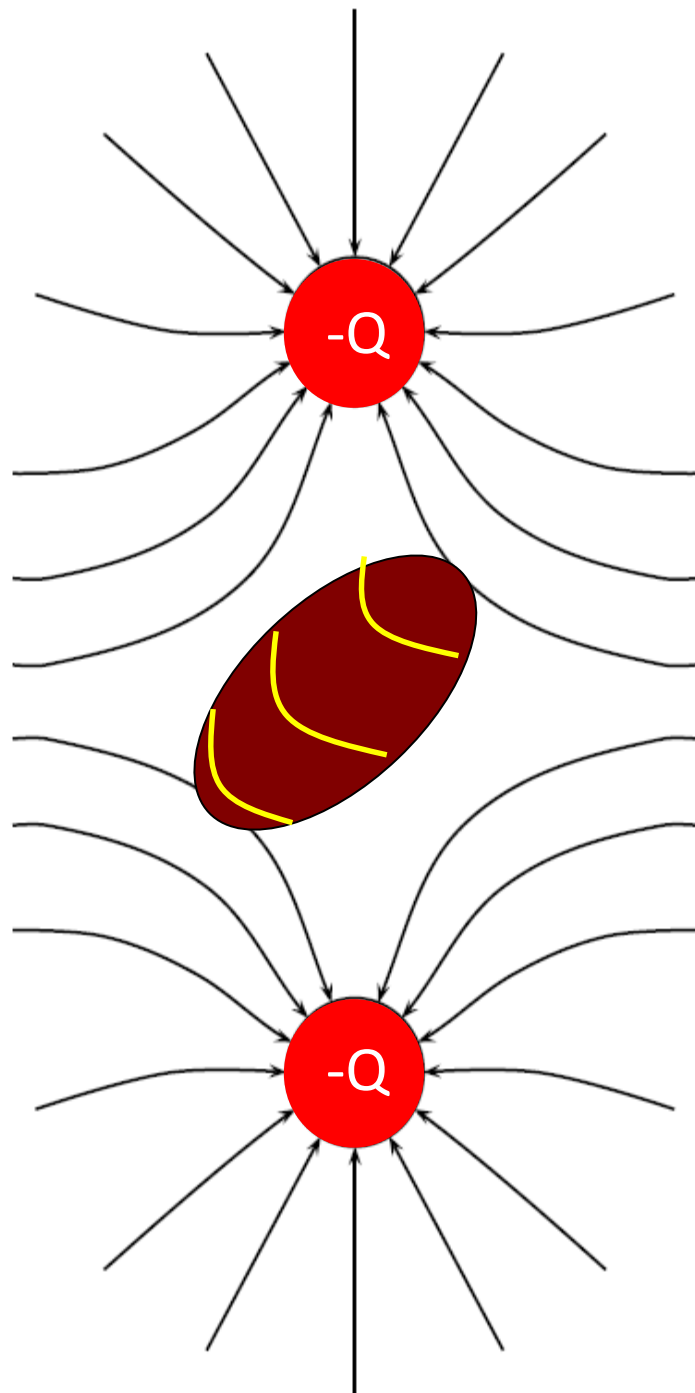
$$\vec{F} = Q\vec{E}$$

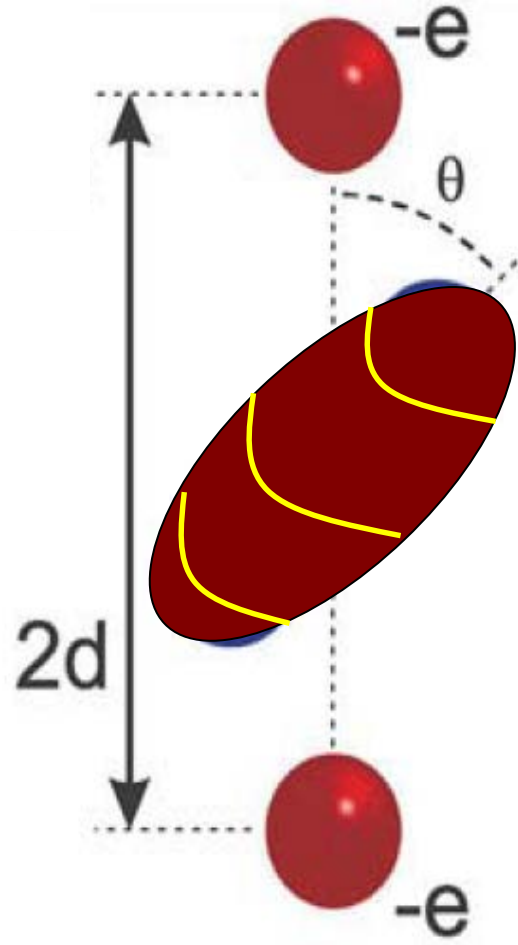
- Force on a general charge:

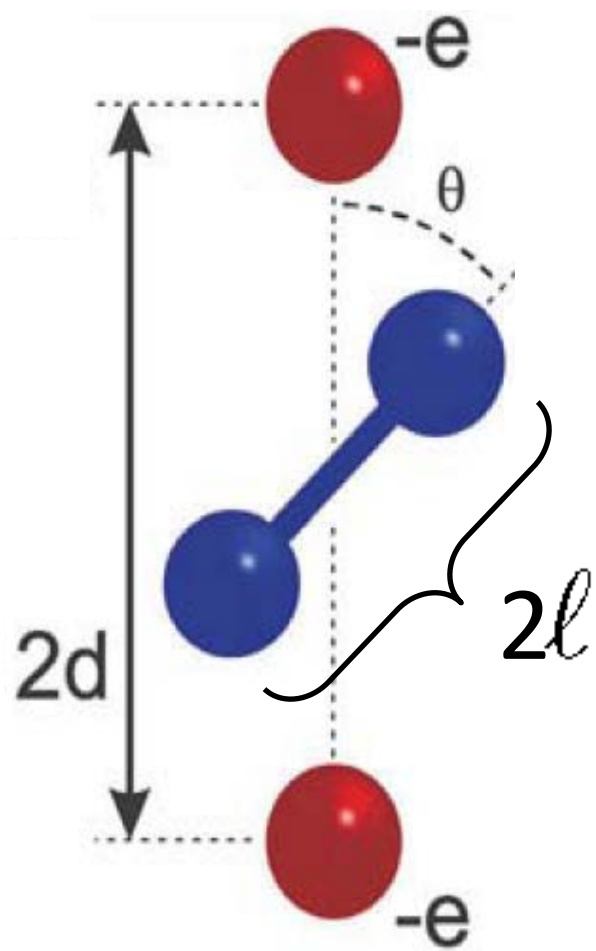
$$\begin{aligned}\vec{F} &= \int \vec{E} dQ \\ &= Q\vec{E}\end{aligned}$$

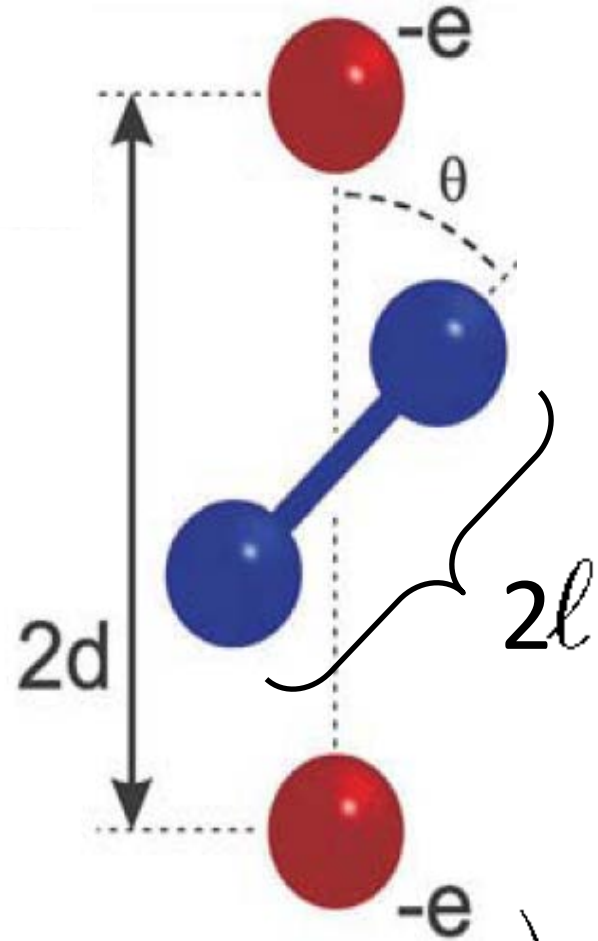
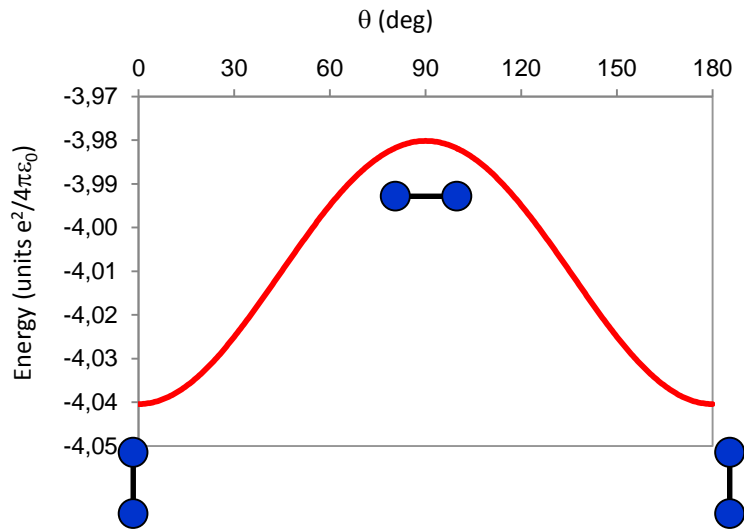






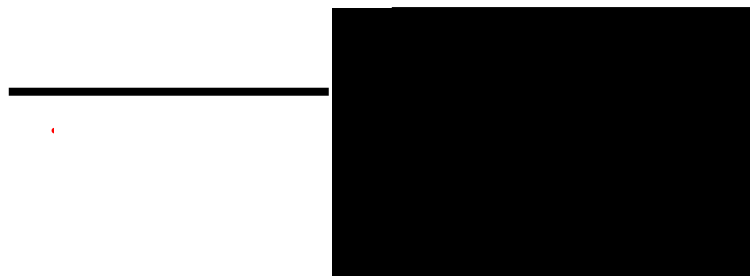
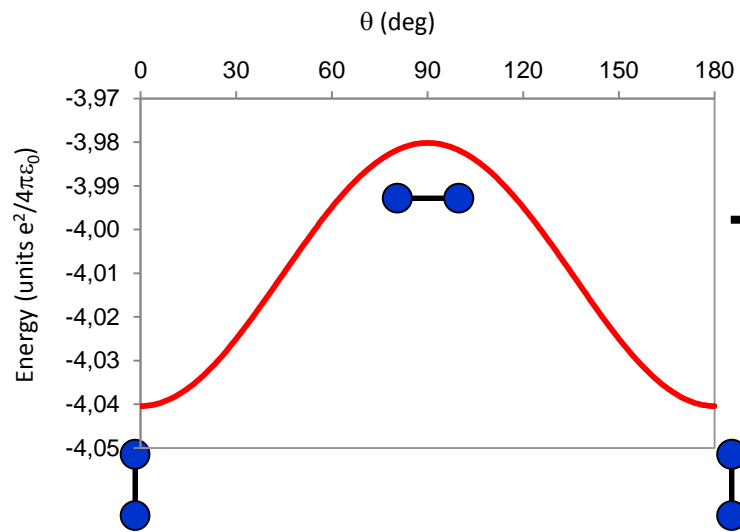


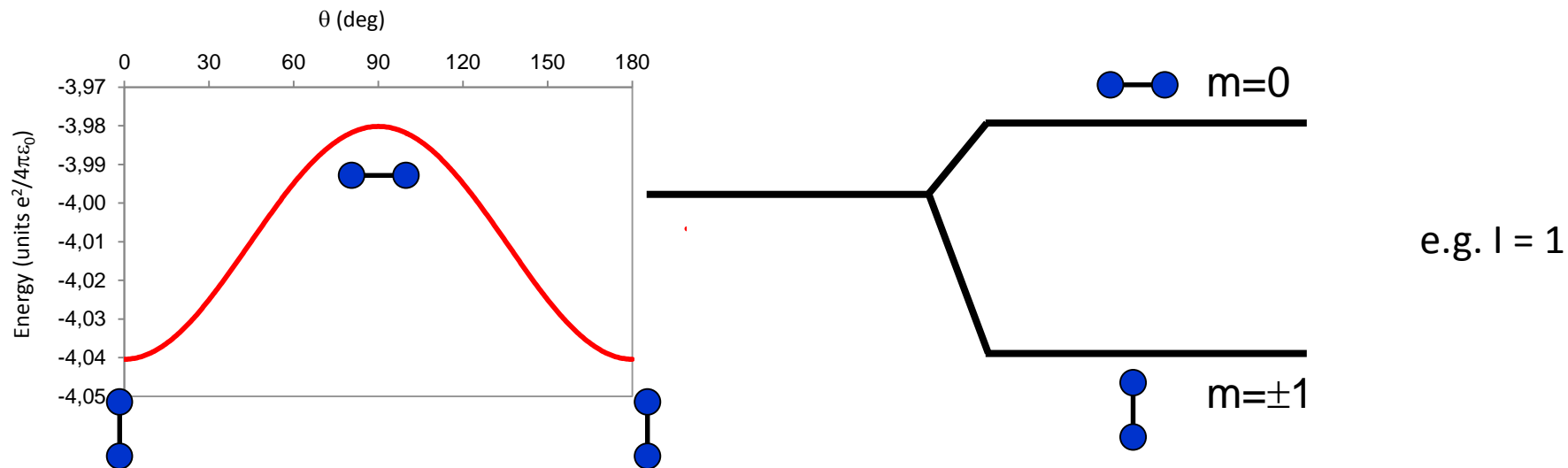




$$C = e^2/(4\pi\epsilon_0)$$

$$E_0(\theta) = -2C \left(\frac{1}{\sqrt{l^2 \sin^2 \theta + (d - l \cos \theta)^2}} + \frac{1}{\sqrt{l^2 \sin^2 \theta + (d + l \cos \theta)^2}} \right)$$

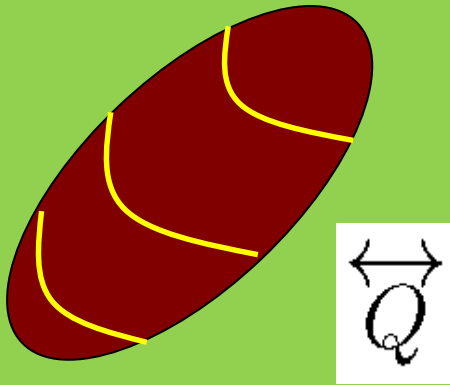




$$H_{qq}^{nuc} = \frac{eQV_{zz}}{4I(2I-1)\hbar^2} \left[(3I_z^2 - I^2) + \frac{\eta}{2} (I_+^2 + I_-^2) \right]$$

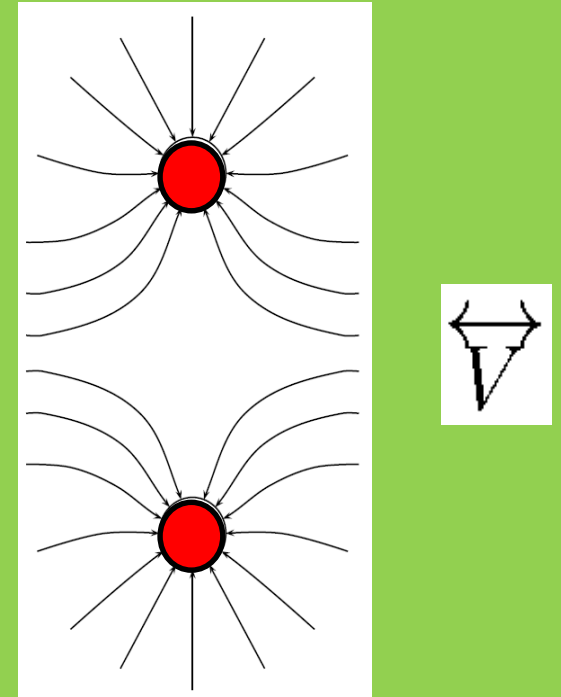
nuclear property

(tensor – rank 2)



electron property

(tensor – rank 2)



interaction energy (dot product) :

$$E_Q \propto \overleftrightarrow{Q} \cdot \overleftrightarrow{V}$$



Electric field gradients (EFG)



- Nuclei with a nuclear quantum number $I \geq 1$ have an electrical quadrupole moment Q
- Nuclear quadrupole interaction (NQI) can aid to determine the distribution of the electronic charge surrounding such a nuclear site
- Experiments

- NMR
- NQR
- Mössbauer
- PAC

$$\nu \approx eQ\Phi / h$$

Nuclear

electronic

Φ EFG traceless tensor

$$\Phi_{ij} = V_{ij} - \frac{1}{3} \delta_{ij} \nabla^2 V$$

$$V_{ij} = \frac{\partial^2 V(0)}{\partial x_i \partial x_j}$$

with

$$V_{xx} + V_{yy} + V_{zz} = 0$$

traceless

$$\begin{vmatrix} V_{aa} & V_{ab} & V_{ac} \\ V_{ba} & V_{bb} & V_{bc} \\ V_{ca} & V_{cb} & V_{cc} \end{vmatrix} \Rightarrow \begin{vmatrix} V_{xx} & 0 & 0 \\ 0 & V_{yy} & 0 \\ 0 & 0 & V_{zz} \end{vmatrix}$$

$$|V_{zz}| \geq |V_{yy}| \geq |V_{xx}|$$

EFG V_{zz}

principal component

$$\eta = \frac{|V_{xx}| - |V_{yy}|}{|V_{zz}|}$$

asymmetry parameter



First-Principles Calculation of the Electric Field Gradient of Li_3N

P. Blaha and K. Schwarz

Institut für Technische Elektrochemie, Technische Universität Wien, A-1060 Vienna, Austria

and

P. Herzig

Institut für Physikalische Chemie, Universität Wien, A-1090 Vienna, Austria

(Received 5 December 1984)

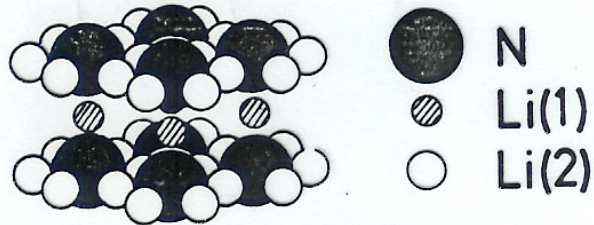


Fig. 1. Crystal structure of Li_3N with increased c dimension

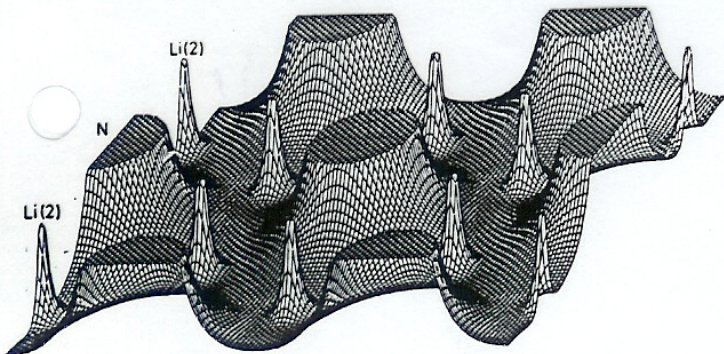


TABLE I. Electric field gradient Φ_{zz} in 10^{20} V m^{-2} .

| Model for Φ_{zz} | Li(1) | Li(2) | Li(1)/Li(2) | N |
|-----------------------|--------|-------|-------------|---------|
| → Point charge | -20.37 | 9.01 | 2.26 | → 0.33 |
| Muffin-tin LAPW | -7.47 | 3.72 | 2.00 | 3.41 |
| → Present work | -6.94 | 3.41 | 2.04 | → 11.16 |
| → Experiment | -5.87 | 2.88 | 2.04 | → 13.04 |

Previous: **point charge model** and **Sternheimer factor** to **experimental value**

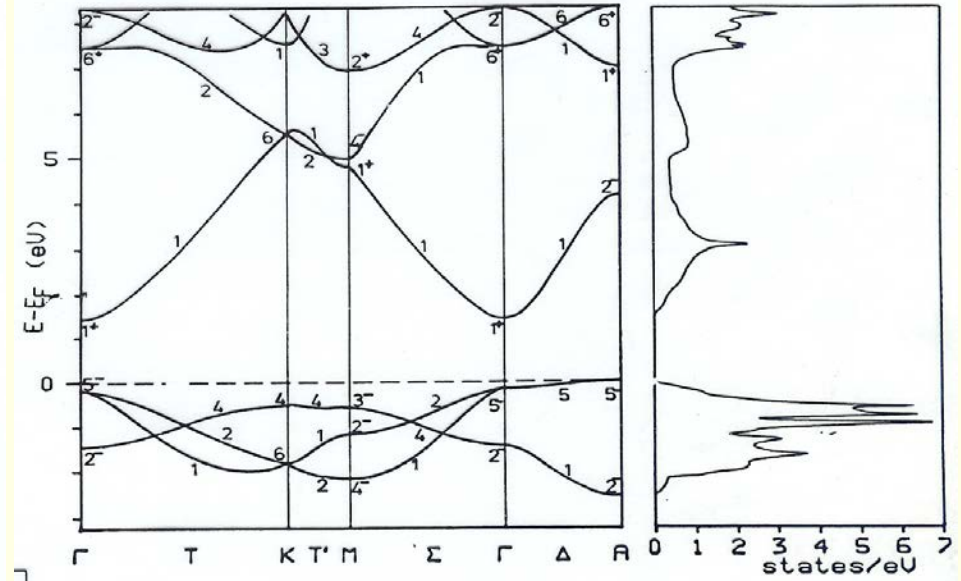
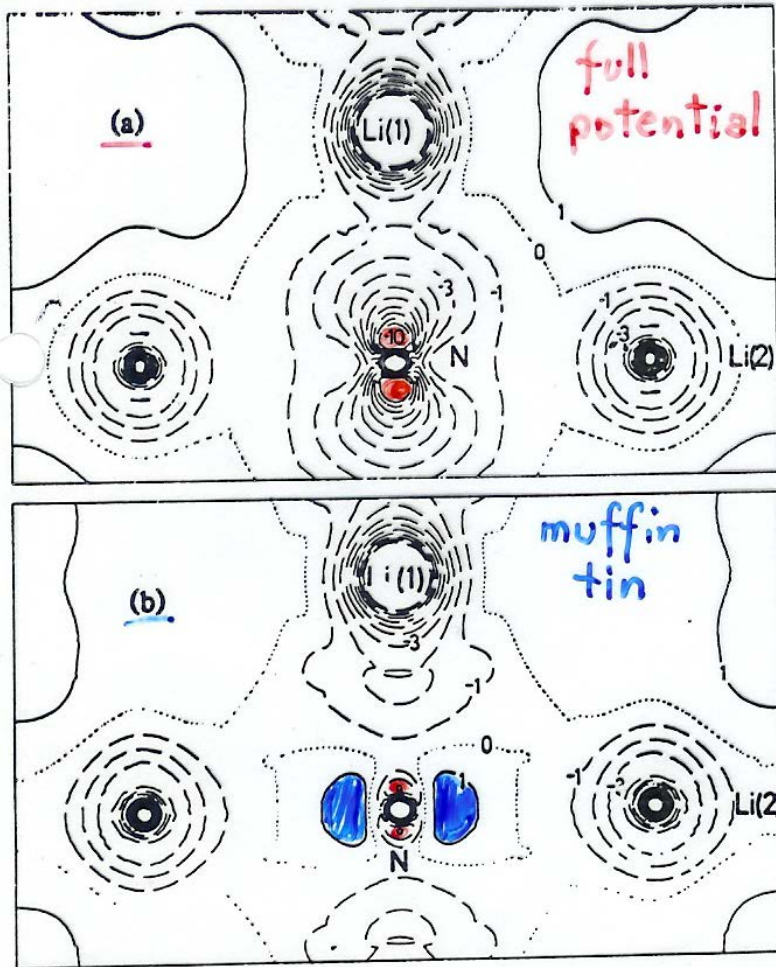


FIG. 1. Difference electron density of Li_3N in the $(1\bar{1}0)$ plane with respect to a superposition of Li^+ and N^{3-} ionic densities; contour intervals and numbers are in units of $0.01 e \text{ \AA}^{-3}$: (a) GP-LAPW (present work), (b) MT-LAPW [taken from Fig. 5(b) of Ref. 10].

- The charge anisotropy around N differs strongly between
 - muffin-tin and
 - full-potential
- affecting the EFG.

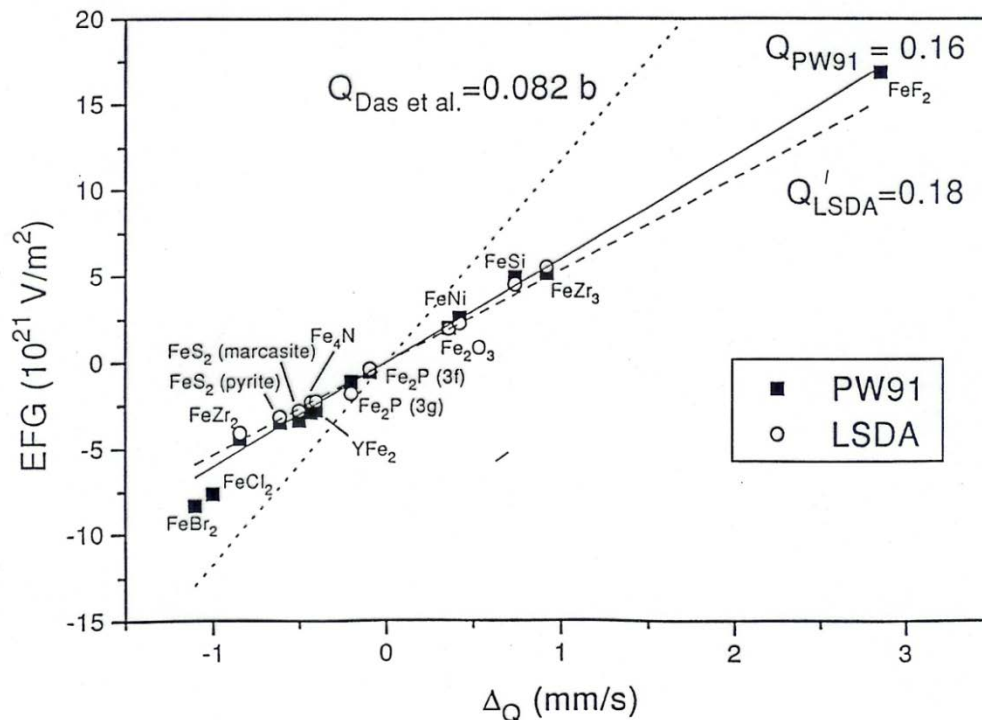
Determination of the Nuclear Quadrupole Moment of ^{57}Fe

Philipp Dufek, Peter Blaha, and Karlheinz Schwarz

Institut für Technische Elektrochemie, Technische Universität Wien, A-1060 Vienna, Austria

(Received 17 July 1995)

Theoretical and experimental Fe-EFG in Fe-compounds



- From the **slope** between
 - the *theoretical EFG* and
 - experimental quadrupole splitting Δ_Q (mm/s)*
- the **nuclear quadrupole moment Q** of the most important **Mössbauer nucleus** is found to be about **twice as large ($Q=0.16$ b)** as so far in literature ($Q=0.082$ b)



theoretical EFG calculations:



EFG is a tensor of **second derivatives of V_c** at the nucleus:

$$V_{ij} = \frac{\partial^2 V(0)}{\partial x_i \partial x_j}$$

$$V_c(r) = \int \frac{\rho(r')}{r-r'} dr' = \sum_{LM} V_{LM}(r) Y_{LM}(\hat{r})$$

Cartesian LM-repr.

$$V_{zz} \propto V_{20}(r=0)$$

$$V_{yy} \propto -\frac{1}{2} V_{20} - V_{22}$$

$$V_{xx} \propto -\frac{1}{2} V_{20} + V_{22}$$

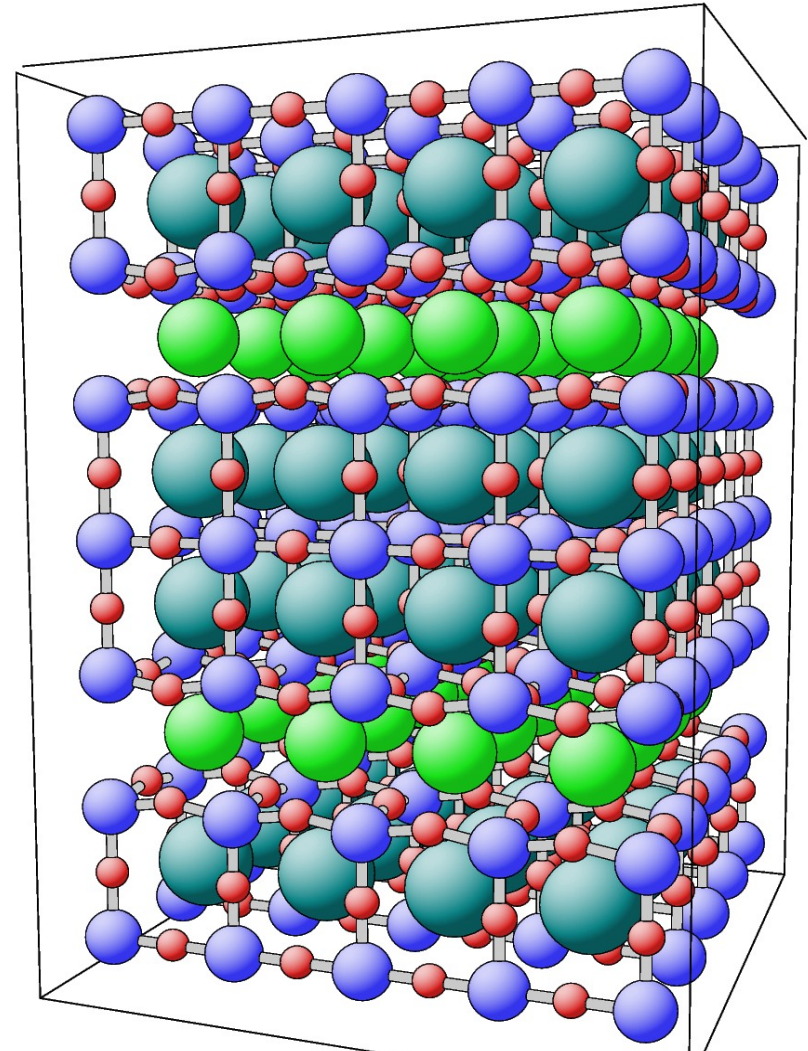
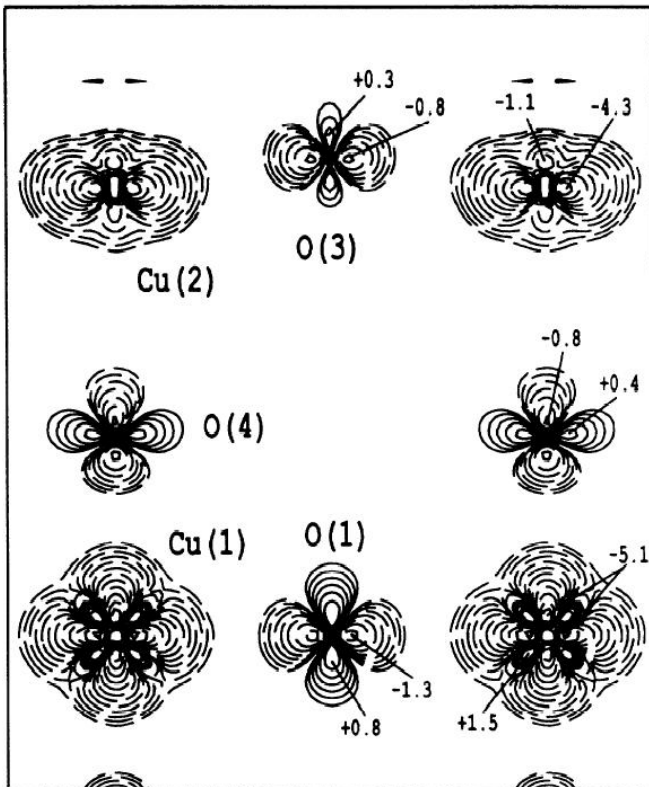
$$V_{zz} \propto \int \frac{\rho(r) Y_{20}}{r^3} dr = V_{zz}^p + V_{zz}^d$$

$$V_{zz}^p \propto \left\langle \frac{1}{r^3} \right\rangle_p \left[\frac{1}{2} (p_x + p_y) - p_z \right]$$

$$V_{zz}^d \propto \left\langle \frac{1}{r^3} \right\rangle_d \left[d_{xy} + d_{x^2-y^2} - \frac{1}{2} (d_{xz} + d_{yz}) - d_{z^2} \right]$$

EFG is proportional to differences in orbital occupations

- $\text{YBa}_2\text{Cu}_3\text{O}_7$
 - Electronic structure
 - Charge density, EFG
- EFG (electric field gradient)



K.Schwarz, C.Ambrosch-Draxl, P.Blaha,
Phys.Rev. B 42, 2051 (1990)

- Interpretation of the EFG (measured by NQR) at the oxygen sites

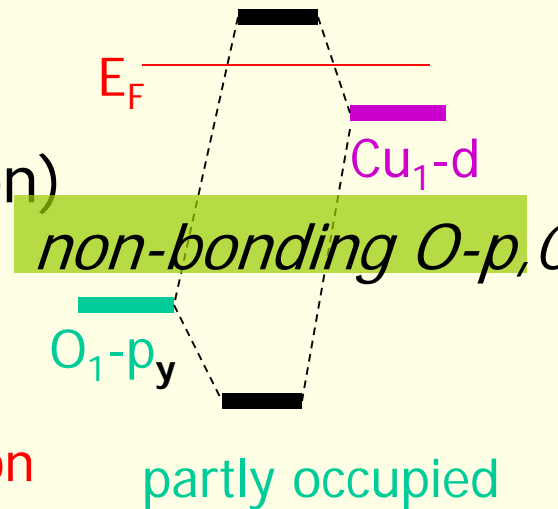
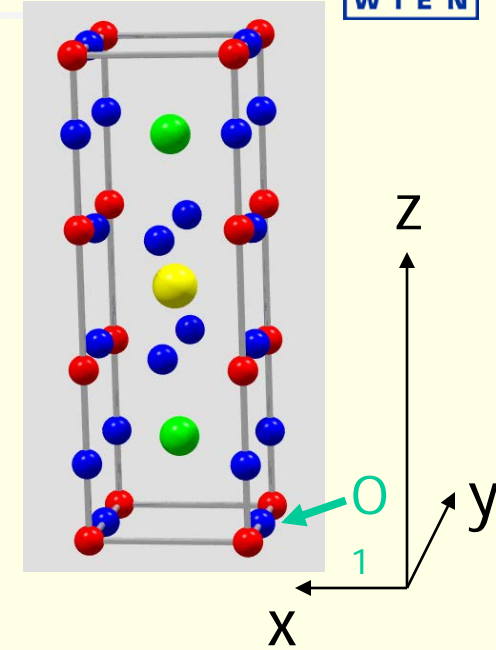
| | p_x | p_y | p_z | V_{aa} | V_{bb} | V_{cc} |
|------|-------------|-------------|-------------|-------------|-------------|-------------|
| O(1) | 1.18 | 0.91 | 1.25 | -6.1 | 18.3 | -12.2 |
| O(2) | 1.01 | 1.21 | 1.18 | 11.8 | -7.0 | -4.8 |
| O(3) | 1.21 | 1.00 | 1.18 | -7.0 | 11.9 | -4.9 |
| O(4) | 1.18 | 1.19 | 0.99 | -4.7 | -7.0 | 11.7 |

Asymmetry count

$$\Delta n_p = p_z - \frac{1}{2}(p_x + p_y)$$

EFG (p-contribution)

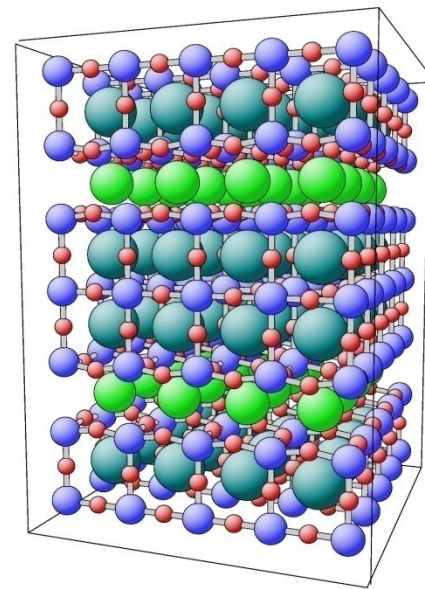
$$V_{zz}^p \propto \Delta n_p \left\langle \frac{1}{r^3} \right\rangle_p$$



EFG is proportional to **asymmetric charge distribution** around given nucleus

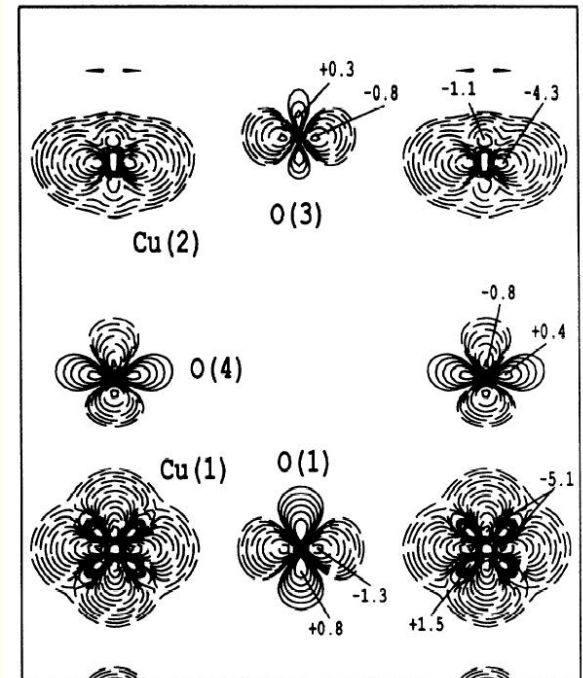


EFG (10^{21} V/m²) in $\text{YBa}_2\text{Cu}_3\text{O}_7$



| Site | | Vxx | Vyy | Vzz | η |
|-------|--------|------|------|-------|--------|
| Y | theory | -0.9 | 2.9 | -2.0 | 0.4 |
| | exp. | - | - | - | - |
| Ba | theory | -8.7 | -1.0 | 9.7 | 0.8 |
| | exp. | 8.4 | 0.3 | 8.7 | 0.9 |
| Cu(1) | theory | -5.2 | 6.6 | -1.5 | 0.6 |
| | exp. | 7.4 | 7.5 | 0.1 | 1.0 |
| Cu(2) | theory | 2.6 | 2.4 | -5.0 | 0.0 |
| | exp. | 6.2 | 6.2 | 12.3 | 0.0 |
| O(1) | theory | -5.7 | 17.9 | -12.2 | 0.4 |
| | exp. | 6.1 | 17.3 | 12.1 | 0.3 |
| O(2) | theory | 12.3 | -7.5 | -4.8 | 0.2 |
| | exp. | 10.5 | 6.3 | 4.1 | 0.2 |
| O(3) | theory | -7.5 | 12.5 | -5.0 | 0.2 |
| | exp. | 6.3 | 10.2 | 3.9 | 0.2 |
| O(4) | theory | -4.7 | -7.1 | 11.8 | 0.2 |
| | exp. | 4.0 | 7.6 | 11.6 | 0.3 |

standard LDA calculations give good EFGs for all sites except Cu(2)



- K.Schwarz, C.Ambrosch-Draxl, P.Blaha, Phys.Rev. B42, 2051 (1990)
- D.J.Singh, K.Schwarz, P.Blaha, Phys.Rev. B46, 5849 (1992)



Cu partial charges in $\text{YBa}_2\text{Cu}_3\text{O}_7$



| | p_x | p_y | p_z | d_{z^2} | $d_{x^2-y^2}$ | d_{xy} | d_{xz} | d_{yz} |
|-------|-------|-------|-------|-----------|---------------|----------|----------|----------|
| Cu(1) | 0.03 | 0.07 | 0.10 | 1.41 | 1.65 | 1.84 | 1.84 | 1.86 |
| Cu(2) | 0.07 | 0.07 | 0.03 | 1.76 | 1.44 | 1.85 | 1.82 | 1.82 |

0.07e

$$V_{zz}^p \propto \Delta n_{p_z} \left\langle \frac{1}{r^3} \right\rangle_p$$

$$\Delta n_{p_z} = 1/2(p_x + p_y) - p_z$$

$$V_{zz}^p = 0.038 \times 250 = 9.5 \text{ (} 10^{21} \text{ V/m}^2\text{)}$$

$$V_{zz}^d \propto \Delta n_d \left\langle \frac{1}{r^3} \right\rangle_d$$

$$\Delta n_d = (d_{xy} + d_{x^2-y^2}) - 1/2(d_{xz} + d_{yz}) - d_{z^2}$$

$$V_{zz}^d = -0.288 \times 47 = -13.5$$

a transfer of **0.07 e** into the d_{z^2} would **increase** the EFG from **-5.0** by

$$\underline{V_{zz}^d} = -0.14 \times 47 = \underline{-6.6}$$

bringing it to **-11.6** inclose to the Experimental value (**-12.3** 10^{21} V/m^2)

How to do it in WIEN2k ?

Electric-field gradient

In regular scf file:

:EFGxxx

:ETAxxx

Main directions of the EFG

} 5 degrees
of freedom

Full analysis printed in case.output2
if EFG keyword in case.in2 is put (UG 7.6)
(split into many different contributions)

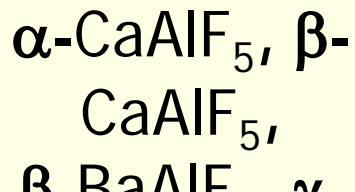
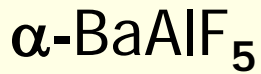
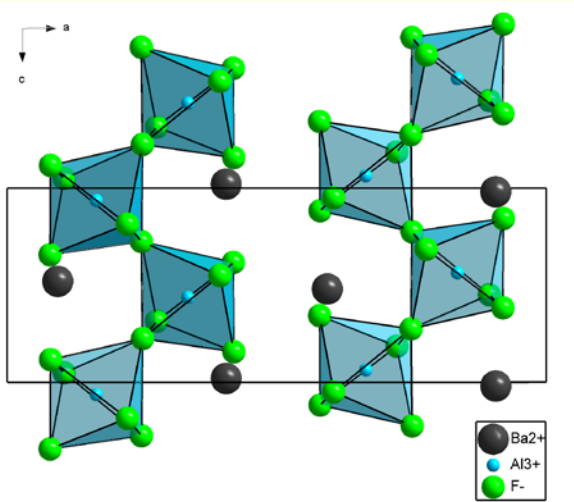
more info:

- Blaha, Schwarz, Dederichs, PRB 37 (1988) 2792
- EFG document in wien2k FAQ (Katrin Koch, SC)

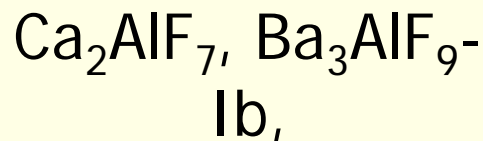
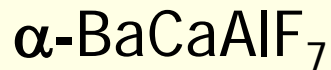
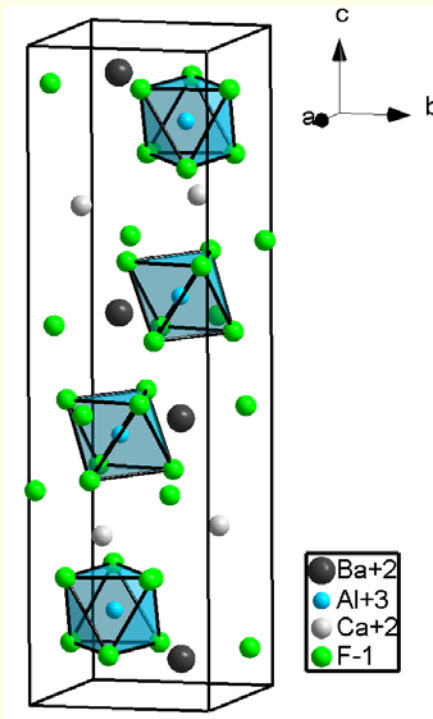
EFGs in fluoroaluminates

10 different phases of known structures from $\text{CaF}_2\text{-AlF}_3$,
 $\text{BaF}_2\text{-AlF}_3$ binary systems and $\text{CaF}_2\text{-BaF}_2\text{-AlF}_3$ ternary system

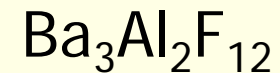
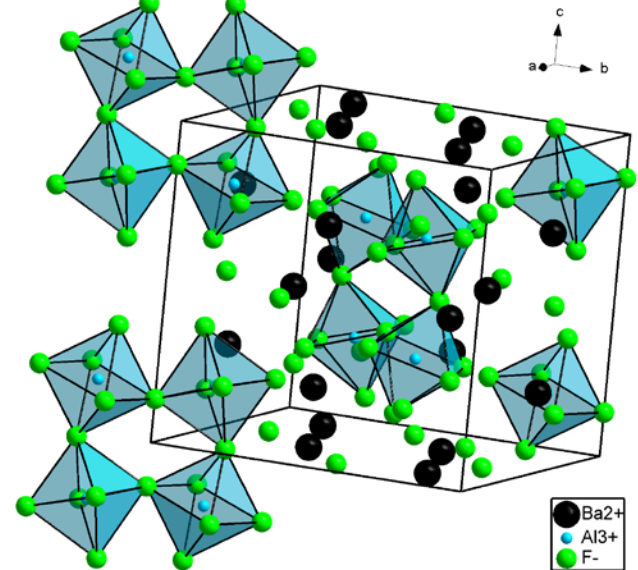
Isolated chains of octahedra linked by corners



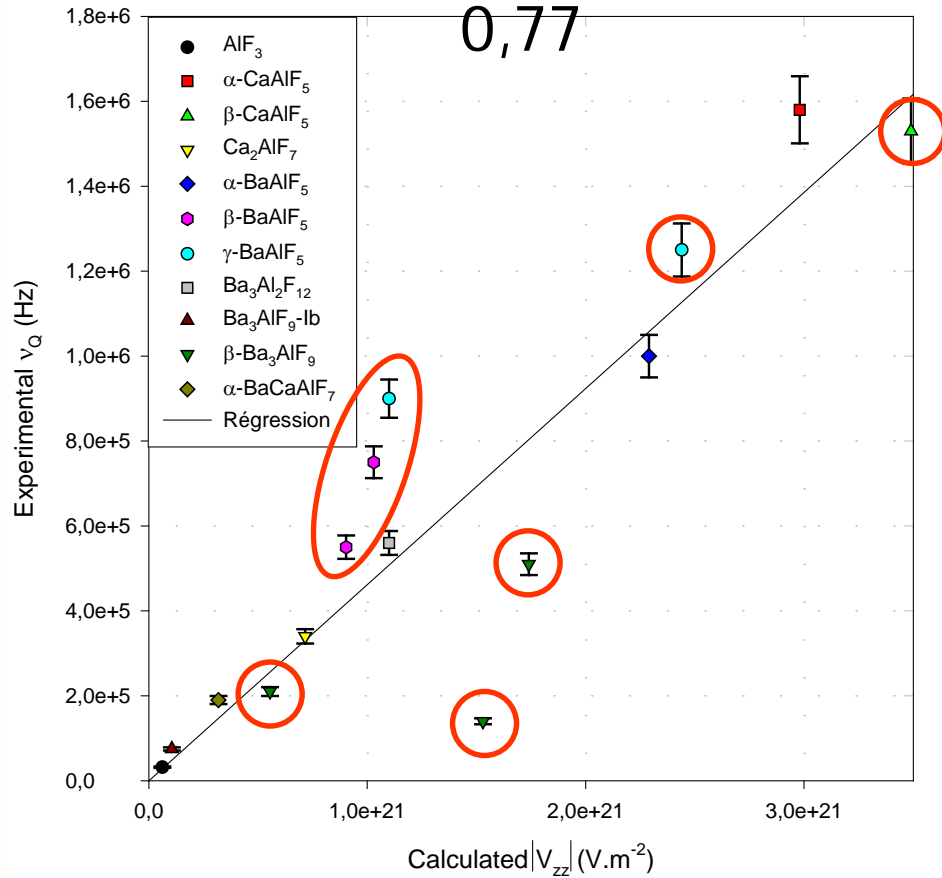
Isolated octahedra



Rings formed by four octahedra sharing corners

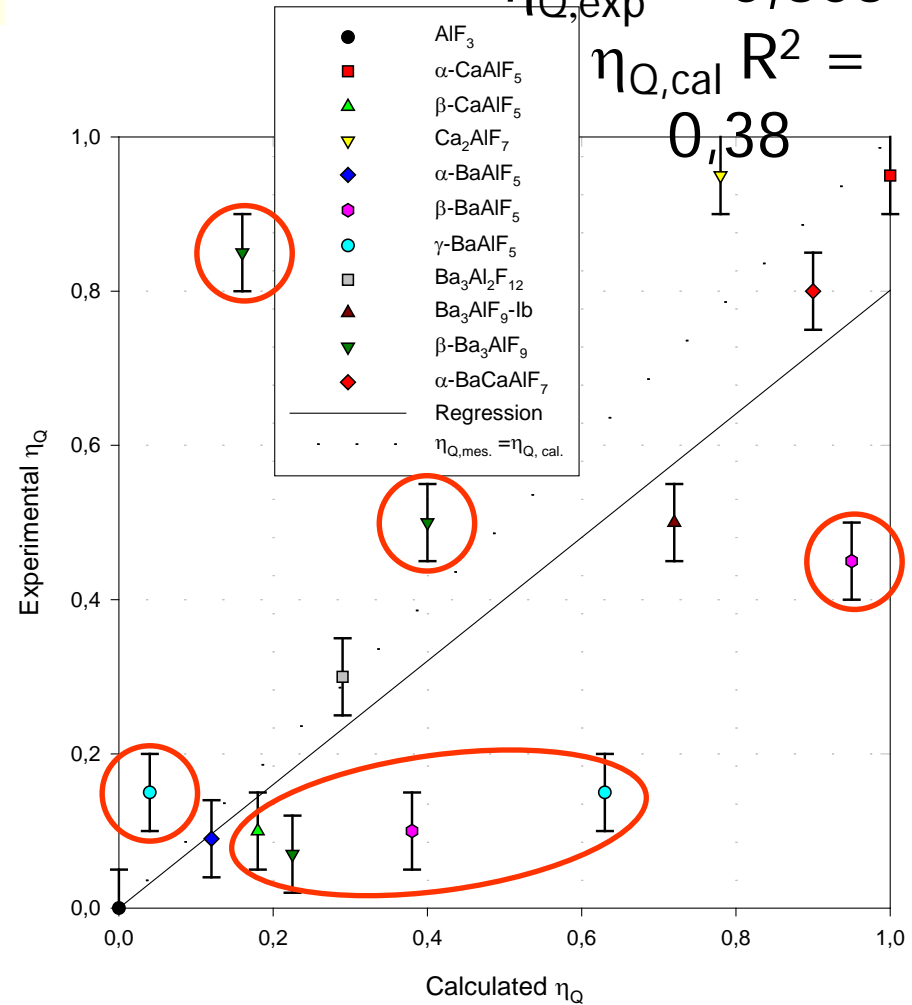


$$\nu_Q = 4,712 \cdot 10^{-16} |V_{zz}| \text{ with } R^2 = 0,77$$



$$\eta_{Q,exp} = 0,803$$

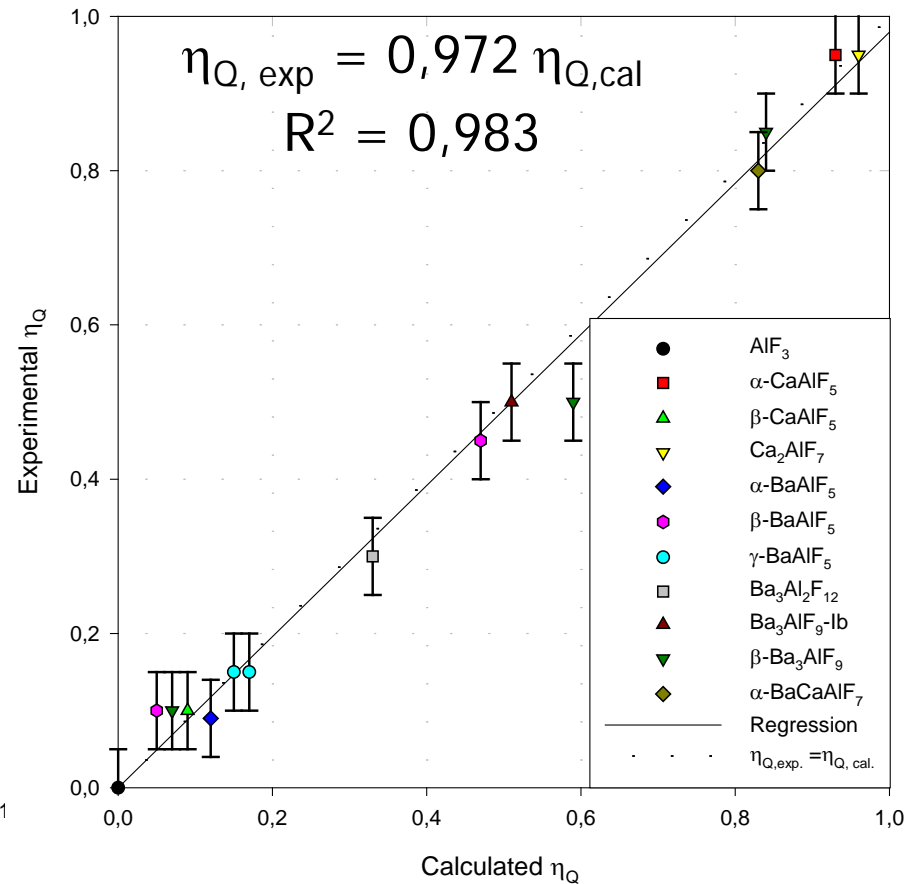
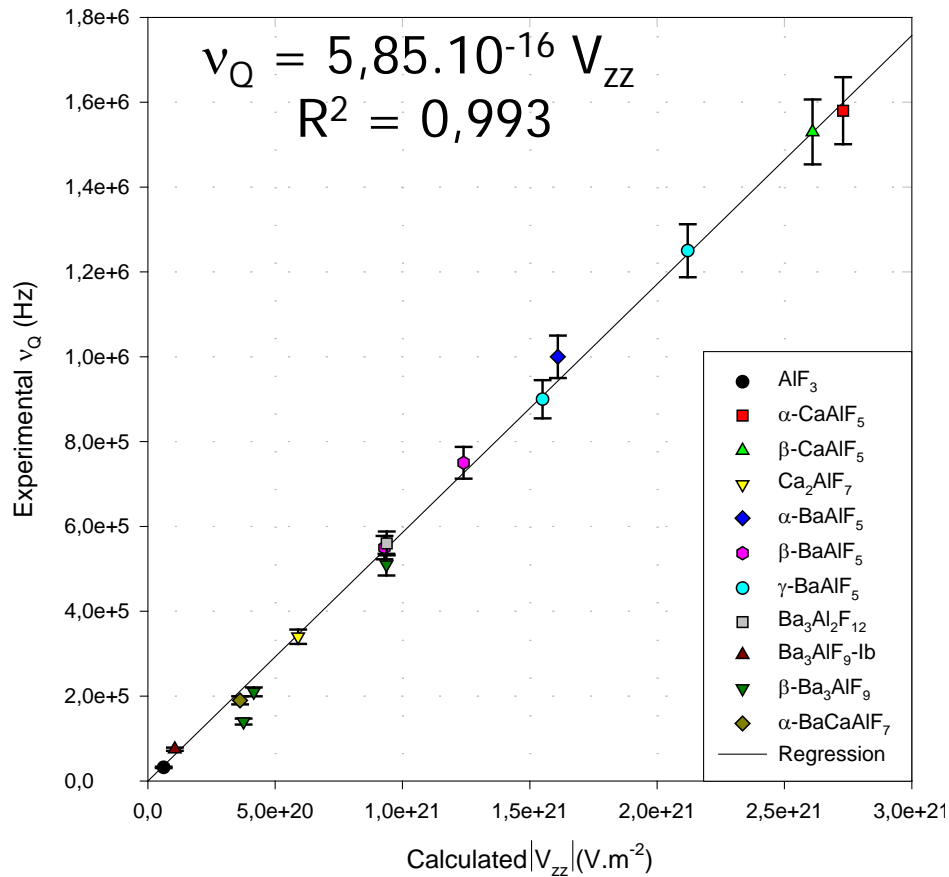
$$\eta_{Q,cal} R^2 = 0,38$$



Important discrepancies when structures are used which were determined from X-ray powder diffraction data

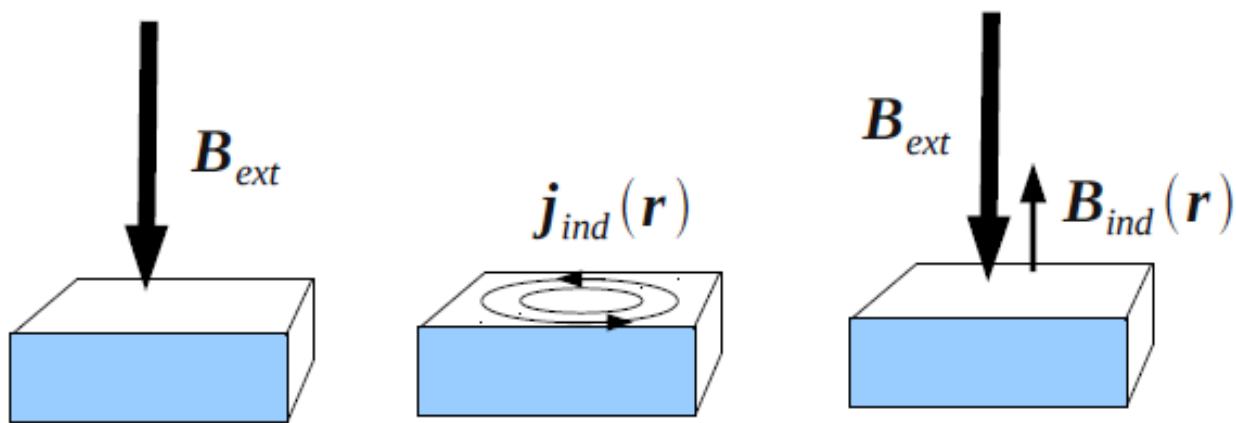


ν_Q and η_Q after structure optimization



Very fine agreement between experimental and calculated values

M.Body, et al., *J.Phys.Chem. A* 2007, 111, 11873
(Univ. LeMans)



$$B_{ext} \rightarrow j_{ind}(\mathbf{r}) \rightarrow B_{ind}(\mathbf{r})$$

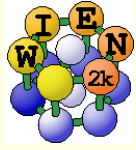
$$B_{ind}(\mathbf{R}) = -\bar{\sigma}(\mathbf{R}) B_{ext}$$

$\sigma(\mathbf{R})$ is the **shielding tensor** at the nucleus \mathbf{R}

chemical shift:
$$\delta(\text{ppm}) = \frac{\sigma_{ref} - \sigma}{\sigma_{ref}} \times 10^6$$

Content

- Definitions
- magnetic hyperfine interaction
- electric quadrupole interaction
- isomer shift
- summary





YBaFe₂O₅ HFF, IS and EFG with GGA+U, LDA/GGA



TABLE VIII: Hyperfine fields B (in Tesla), isomer shifts δ (mm/s) and quadrupole coupling constants eQV_{zz} (mm/s) for the CO phase for various exchange and correlation potentials and experiment⁸⁻¹⁰.

| CO | U_{eff} [eV] | exp. | GGA+U | | | | LDA | GGA |
|------------------|----------------|-------------|--------|--------|--------|--------|-------|--------|
| | | | 5 | 6 | 7 | 8 | | |
| Fe ²⁺ | B_{dip} | — | -16.29 | -16.49 | -16.66 | -16.83 | -6.68 | -12.67 |
| | B_{orb} | — | -6.73 | -6.90 | -8.26 | -7.65 | -9.57 | -6.34 |
| | $B_{contact}$ | — | 32.25 | 32.23 | 32.58 | 32.60 | 32.21 | 31.58 |
| | B_{tot} | ~ 8 | 9.23 | 8.83 | 7.66 | 8.13 | 15.96 | 12.57 |
| | δ | ~ 1 | 0.92 | 0.94 | 0.96 | 0.99 | 0.74 | 0.79 |
| | eQV_{zz} | $3.6 - 4^a$ | 3.66 | 3.74 | 3.81 | 3.89 | -0.82 | 2.60 |
| Fe ³⁺ | B_{dip} | — | -0.67 | -0.60 | -0.52 | -0.45 | 1.29 | 0.39 |
| | B_{orb} | — | -0.52 | -0.45 | -0.37 | -0.28 | -7.96 | -2.65 |
| | $B_{contact}$ | — | 37.65 | 38.28 | 38.15 | 37.86 | 29.64 | 31.63 |
| | B_{tot} | ~ 50 | 36.46 | 37.24 | 37.26 | 37.12 | 22.97 | 29.37 |
| | δ | ~ 0.4 | 0.33 | 0.30 | 0.28 | 0.25 | 0.50 | 0.47 |
| | eQV_{zz} | $1 - 1.5^a$ | 1.46 | 1.50 | 1.51 | 1.52 | 1.04 | -0.30 |

^adepending on rare earth ion

| VM | U_{eff} [eV] | exp. | GGA+U | | | | LDA | GGA |
|--------------------|----------------|------------|-------|-------|-------|-------|-------|-------|
| | | | 5 | 6 | 7 | 8 | | |
| Fe ^{2.5+} | B_{dip} | — | -3.00 | -2.98 | -2.95 | -2.87 | -2.13 | -2.83 |
| | B_{orb} | — | -3.11 | -2.99 | -2.84 | -2.74 | -5.47 | -4.56 |
| | $B_{contact}$ | — | 41.17 | 40.96 | 41.45 | 41.17 | 33.10 | 36.36 |
| | B_{tot} | ~ 30 | 35.06 | 34.98 | 35.67 | 35.56 | 25.50 | 28.98 |
| | δ | ~ 0.5 | 0.53 | 0.52 | 0.51 | 0.49 | 0.60 | 0.60 |
| | eQV_{zz} | ~ 0.1 | 0.12 | 0.13 | 0.13 | 0.13 | 0.19 | -0.27 |



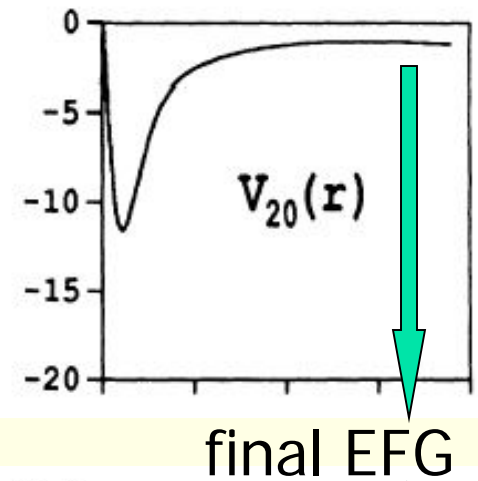
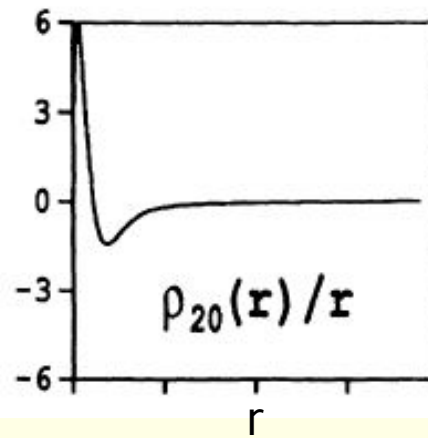
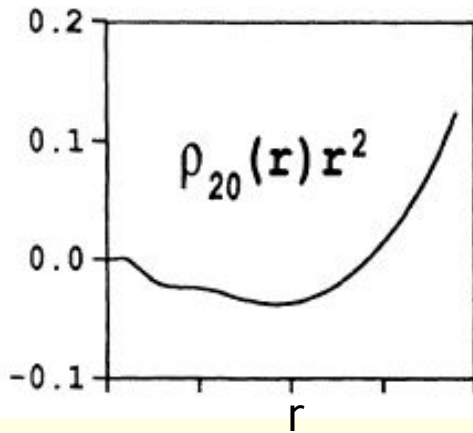
Cu(2) and O(4) EFG as function of r



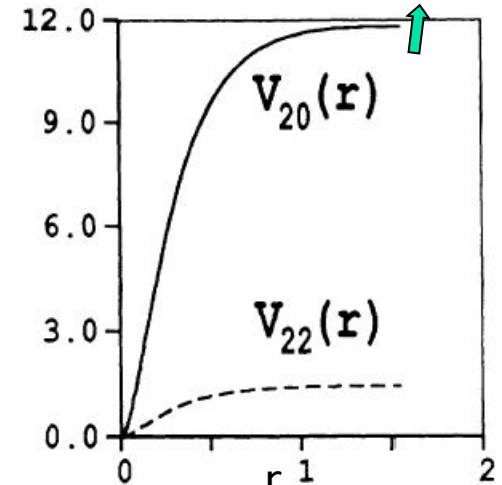
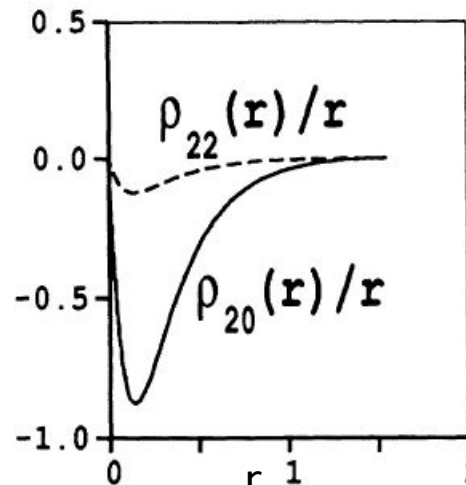
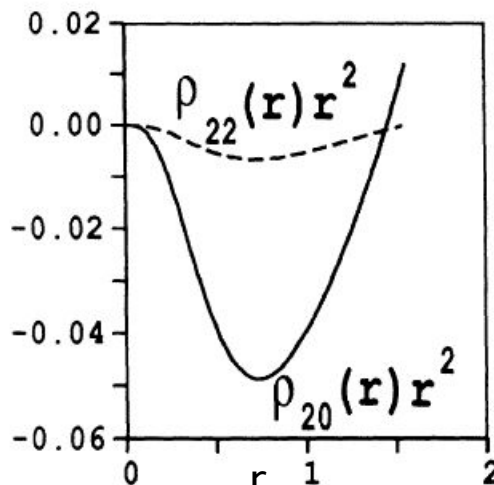
- EFG is determined by the non-spherical charge density inside sphere

$$\rho(r) = \sum_{LM} \rho_{LM}(r) Y_{LM} \quad V_{zz} \propto \int \frac{\rho(r) Y_{20}}{r^3} dr = \int \rho_{20}(r) r dr$$

- Cu(2)

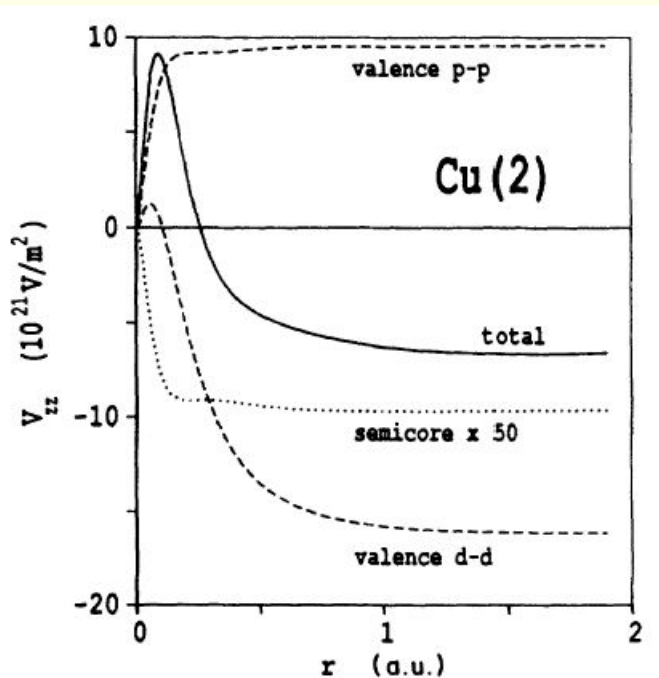


- O(4)





- Depending on the atom, the main EFG-contributions come from anisotropies (in occupation or wave function)
 - *semicore p-states* (eg. *Ti 3p* much more important than *Cu 3p*)
 - *valence p-states* (eg. *O 2p* or *Cu 4p*)
 - *valence d-states* (eg. *TM 3d,4d,5d* states; in metals "small")
 - *valence f-states* (only for "localized" *4f,5f* systems)



usually only contributions within the first node or within 1 bohr are important.

