

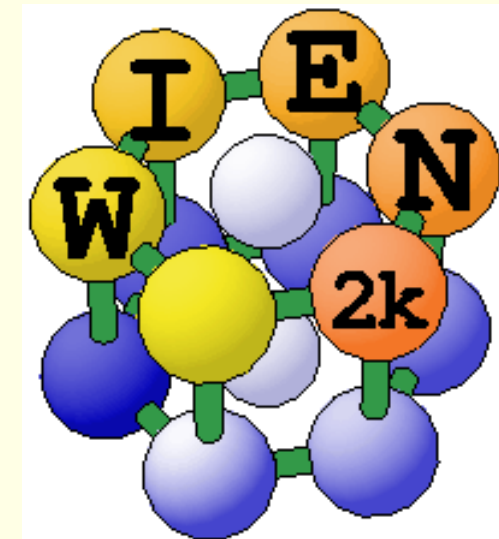
Hyperfine interactions

Mössbauer, PAC and NMR Spectroscopy:
Quadrupole splittings, Isomer shifts, Hyperfine fields (~~NMR shifts~~)

Peter Blaha

Institute of Materials Chemistry

TU Wien





Definition of Hyperfine Interactions



hyperfine interaction

=

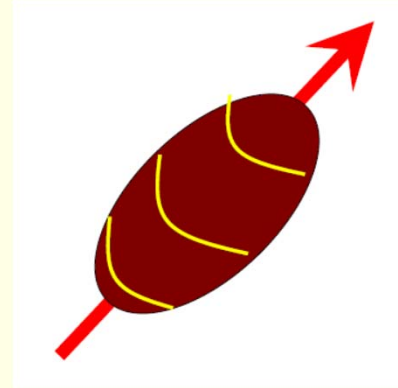
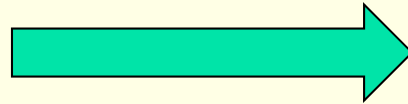
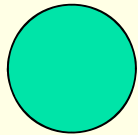
all aspects of the nucleus-electron interaction which go beyond an electric point charge for a nucleus

and is

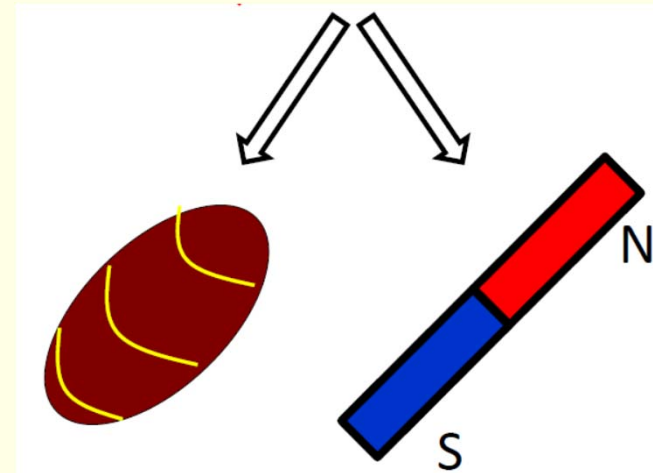
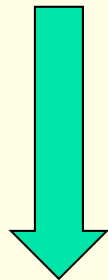
**measured at the nucleus
(affects the nucleus)**



description of the nucleus:



electric point charge (Z/r)



nucleus with volume, shape and magnetic moment



How to measure hyperfine interactions ?



- **NMR**
- **NQR**
- **Mössbauer spectroscopy**
- **TDPAC**





Electric Hyperfine-Interaction



- between nuclear charge distribution (σ) and external potential

$$E = \int \sigma_n(x) V(x) dx$$

- Taylor-expansion at the nuclear position

$$E = V_0 Z$$

direction independent constant
(monopole interaction)

$$+ \sum_i \frac{\partial V(0)}{\partial x_i} \int \sigma(x) x_i dx$$

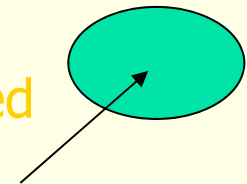
electric field \times
nuclear dipol moment (=0)

$$+ \frac{1}{2} \sum_{ij} \frac{\partial^2 V(0)}{\partial x_i \partial x_j} \iint \sigma(x) x_i x_j dx$$

electric fieldgradient \times
nuclear quadrupol moment Q

+ ...

higher terms neglected



nucleus with charge Z , but not a sphere



Electric monopole interaction



■ Mössbauer Isomer Shift δ :

- *integral over nuclear radius of electron density \times nuclear charge*
- *nuclear radii are different for ground and excited state*

$$\delta = \Delta E_A - \Delta E_Q$$

$$= (\rho_A(0) - \rho_Q(0))(R_A^2 - R_Q^2)$$

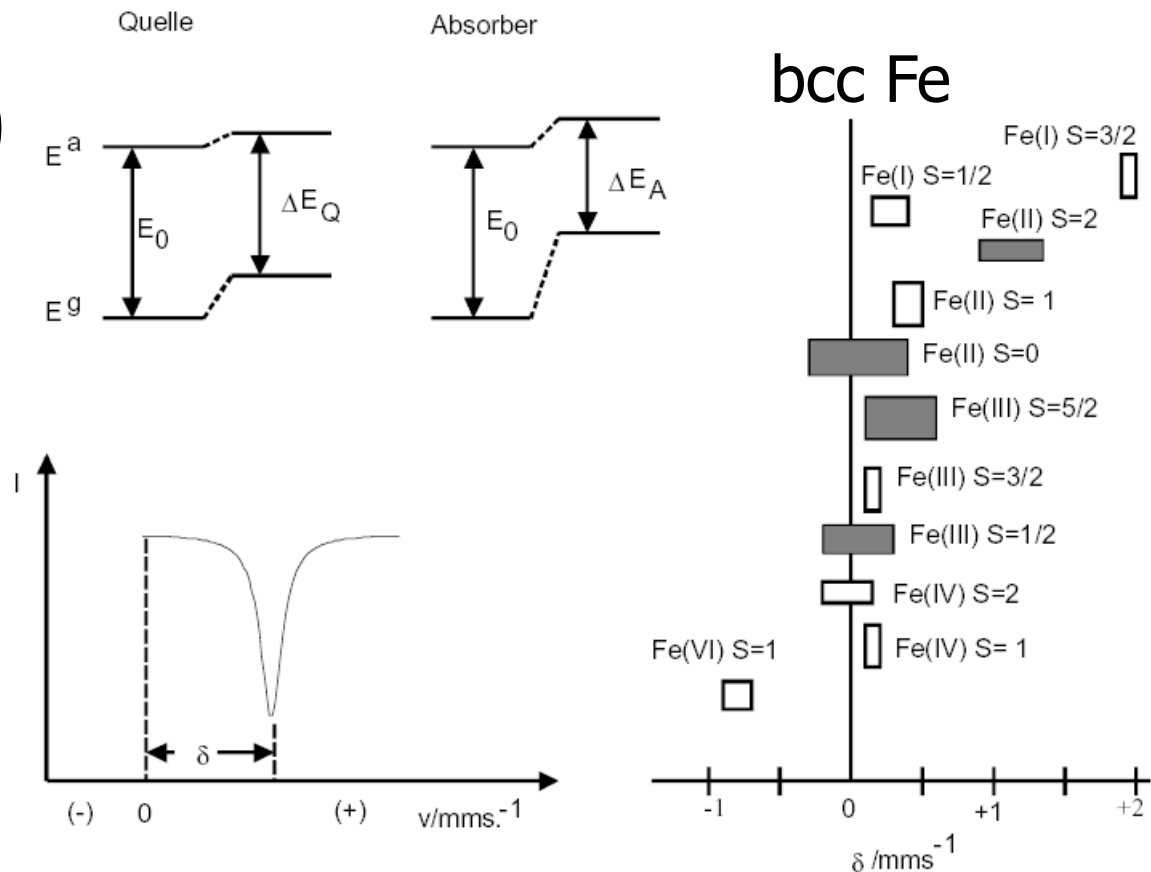
$$= \alpha(\rho_A(0) - \rho_Q(0))$$

$$\alpha(^{57}\text{Fe}) = -0.24 \text{ mm/s } a_0^3$$

\rightarrow large $\rho_A(0) \rightarrow$ neg. IS

$\text{Fe}^{2+}, \text{Fe}^{3+}, \dots$

$$:RTOxxx = \rho(0)$$





Quadrupole interaction



$$E = \frac{1}{2} \sum_{ij} V_{ij} Q$$

$Q(^{57}\text{Fe}) = 0.16$ barn

V_{ij} : **traceless** 3x3 tensor of **electric field gradient EFG** (2nd derivative of $V(0)$)

similarity transformation

$$\begin{pmatrix} V_{aa} & V_{ab} & V_{ac} \\ V_{ba} & V_{bb} & V_{bc} \\ V_{ca} & V_{cb} & V_{cc} \end{pmatrix} \Rightarrow \begin{pmatrix} V_{xx} & 0 & 0 \\ 0 & V_{yy} & 0 \\ 0 & 0 & V_{zz} \end{pmatrix}$$

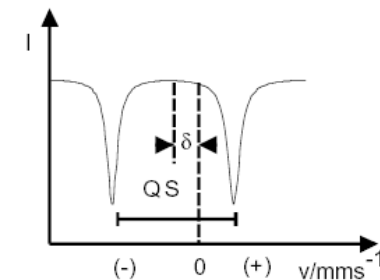
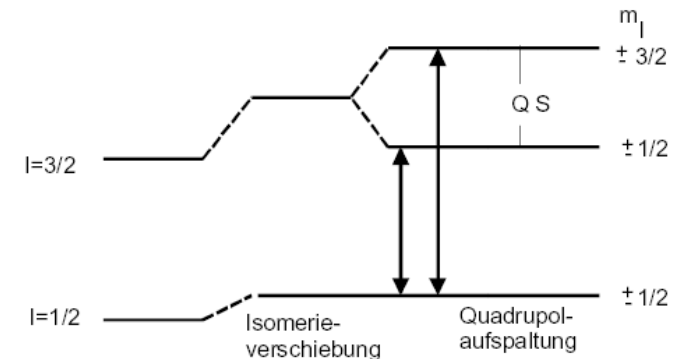
with $V_{xx} + V_{yy} + V_{zz} = 0$
 $|V_{zz}| \geq |V_{yy}| \geq |V_{xx}|$

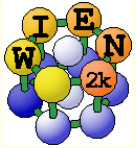
$$V_{ij} = \frac{\partial^2 V(0)}{\partial x_i \partial x_j}$$

EFG characterized by principal component V_{zz} and asymmetry parameter η

$$\eta = \frac{|V_{xx}| - |V_{yy}|}{|V_{zz}|}$$

$$QS = \frac{1}{2} eQV_{zz} \sqrt{1 + \frac{\eta^2}{3}}$$





First-principles calculation of EFG



VOLUME 54, NUMBER 11 , 1192 PHYSICAL REVIEW LETTERS

18 MARCH 1985

First-Principles Calculation of the Electric Field Gradient of Li_3N

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and

P. Herzig

Institut für Physikalische Chemie, Universität Wien, A-1090 Vienna, Austria

(Received 5 December 1984)

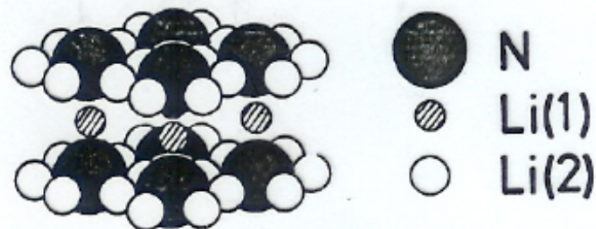


Fig. 1. Crystal structure of Li_3N with increased c dimension

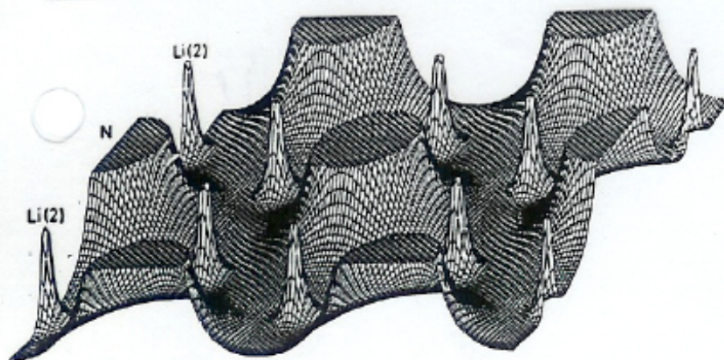
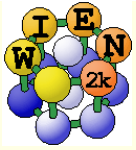


TABLE I. Electric field gradient Φ_{zz} in 10^{20} V m^{-2} .

Model for Φ_{zz}	Li(1)	Li(2)	Li(1)/Li(2)	N
Point charge	-20.37	9.01	2.26	0.33
Muffin-tin LAPW	-7.47	3.72	2.00	3.41
Present work	-6.94	3.41	2.04	11.16
Experiment	-5.87	2.88	2.04	13.04

Previous: **point charge model** and **Sternheimer factor** to **experimental value**



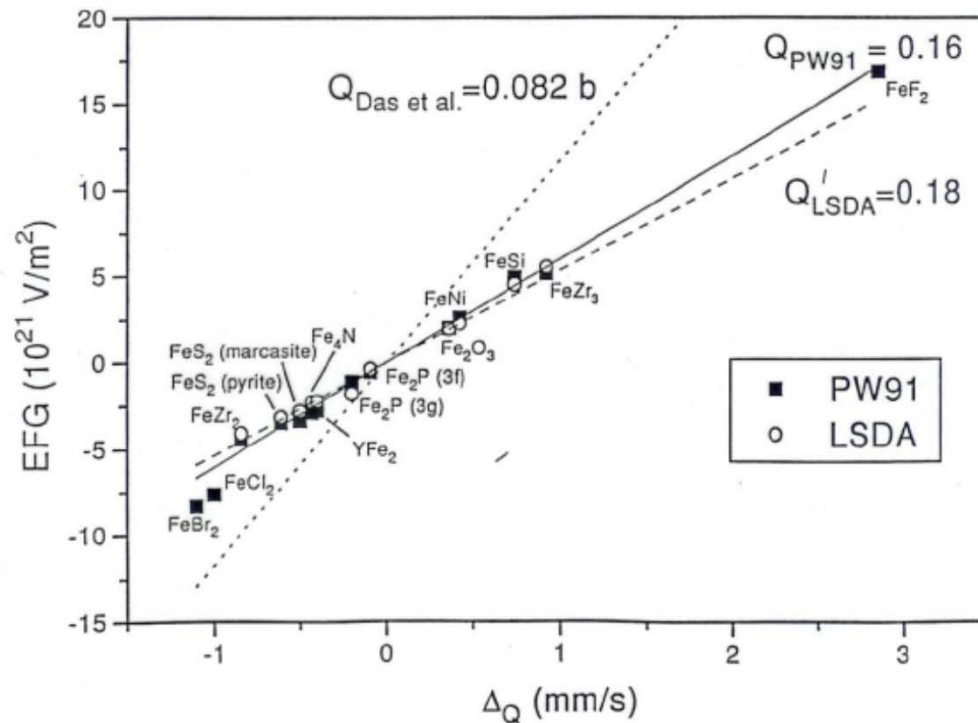
Determination of the Nuclear Quadrupole Moment of ^{57}Fe

Philipp Dufek, Peter Blaha, and Karlheinz Schwarz

Institut für Technische Elektrochemie, Technische Universität Wien, A-1060 Vienna, Austria

(Received 17 July 1995)

Theoretical and experimental Fe-EFG in Fe-compounds



$$E = \frac{1}{2} \sum_{ij} V_{ij} Q$$

- From the **slope** between
 - *the theoretical EFG and*
 - *experimental quadrupole splitting Δ_Q (mm/s)*
- the **nuclear quadrupole moment Q** of the most important **Mössbauer nucleus** is found to be about **twice as large ($Q=0.16$ b)** as so far in literature ($Q=0.082$ b)



theoretical EFG calculations



We write the charge density and the potential inside the atomic spheres in a lattice-harmonics expansion

$$\rho(r) = \sum_{LM} \rho_{LM}(\bar{r}) Y_{LM}(\hat{r}); \quad V_{ij} = \frac{\partial^2 V(0)}{\partial x_i \partial x_j}; \quad V_c(r) = \int \frac{\rho(r')}{r-r'} dr'$$

spatial decomposition:

$$V_{zz} \propto \int \frac{\rho(r) Y_{20}}{r^3} d^3 r = \int_{\text{sphere}} \frac{\sum_{LM} \rho_{LM}(r) Y_{LM} Y_{20}}{r^3} d^3 r + \int_{\text{interstitial}} \frac{\rho(r) Y_{20}}{r^3} d^3 r$$

$$V_{zz} \propto \int_{\text{sphere}} \frac{\rho_{20}(r)}{r^3} dr + \text{interstitial}$$

orbital decomposition:

$$\rho_{20}(r) = \int \sum_{k,n,l,l',m,m'} \phi_{lm}^{nk*} \phi_{l'm'}^{nk} Y_{20} d\hat{r} \Rightarrow p-p; d-d; (s-d) \text{ contr.}$$

$$V_{zz} = V_{zz}^{pp} + V_{zz}^{dd} + \dots + \text{interstitial}$$



theoretical EFG calculations



$$V_{zz} = V_{zz}^{pp} + V_{zz}^{dd} + \dots + \textit{interstitial}$$

$$V_{zz}^{pp} \propto \left\langle \frac{1}{r^3} \right\rangle_p \left[\frac{1}{2} (p_x + p_y) - p_z \right]$$

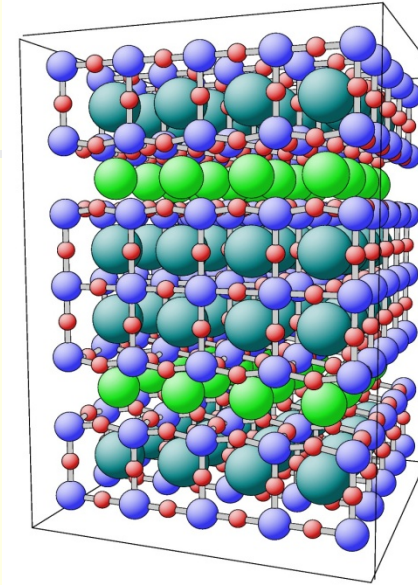
$$V_{zz}^{dd} \propto \left\langle \frac{1}{r^3} \right\rangle_d \left[d_{xy} + d_{x^2-y^2} - \frac{1}{2} (d_{xz} + d_{yz}) - d_{z^2} \right]$$

- EFG is **proportional to differences of orbital occupations**, e.g. between $p_x p_y$ and p_z
- if these occupancies are the same by symmetry (cubic): $\mathbf{V}_{zz} = \mathbf{0}$
- with "axial" (hexagonal, tetragonal) symmetry ($p_x = p_y$): $\eta = \mathbf{0}$

In the following various examples will be presented.



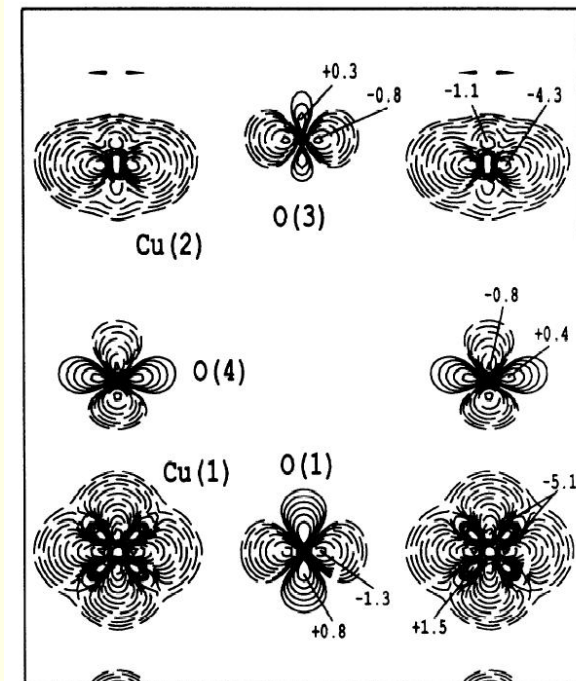
EFG (10^{21} V/m²) in $\text{YBa}_2\text{Cu}_3\text{O}_7$



Site		V _{xx}	V _{yy}	V _{zz}	η
Y	theory	-0.9	2.9	-2.0	0.4
	exp.	-	-	-	-
Ba	theory	-8.7	-1.0	9.7	0.8
	exp.	8.4	0.3	8.7	0.9
Cu(1)	theory	-5.2	6.6	-1.5	0.6
	exp.	7.4	7.5	0.1	1.0
Cu(2)	theory	2.6	2.4	-5.0	0.0
	exp.	6.2	6.2	12.3	0.0
O(1)	theory	-5.7	17.9	-12.2	0.4
	exp.	6.1	17.3	12.1	0.3
O(2)	theory	12.3	-7.5	-4.8	0.2
	exp.	10.5	6.3	4.1	0.2
O(3)	theory	-7.5	12.5	-5.0	0.2
	exp.	6.3	10.2	3.9	0.2
O(4)	theory	-4.7	-7.1	11.8	0.2
	exp.	4.0	7.6	11.6	0.3

standard LDA calculations give good EFGs for all sites except Cu(2)

- K.Schwarz, C.Ambrosch-Draxl, P.Blaha, Phys.Rev. B42, 2051 (1990)
- D.J.Singh, K.Schwarz, K.Schwarz, Phys.Rev. B46, 5849 (1992)





EFG in $\text{YBa}_2\text{Cu}_3\text{O}_7$



■ Interpretation of the EFG at the oxygen sites

	p_x	p_y	p_z	V_{aa}	V_{bb}	V_{cc}
O(1)	1.18	0.91	1.25	-6.1	18.3	-12.2
O(2)	1.01	1.21	1.18	11.8	-7.0	-4.8
O(3)	1.21	1.00	1.18	-7.0	11.9	-4.9
O(4)	1.18	1.19	0.99	-4.7	-7.0	11.7

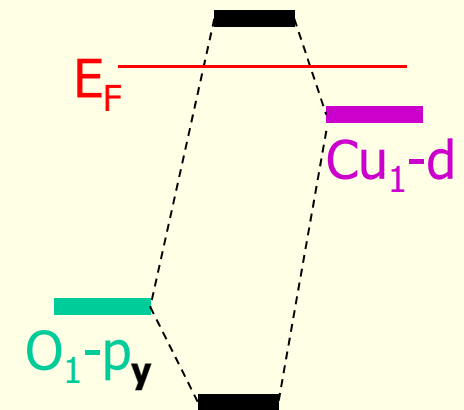
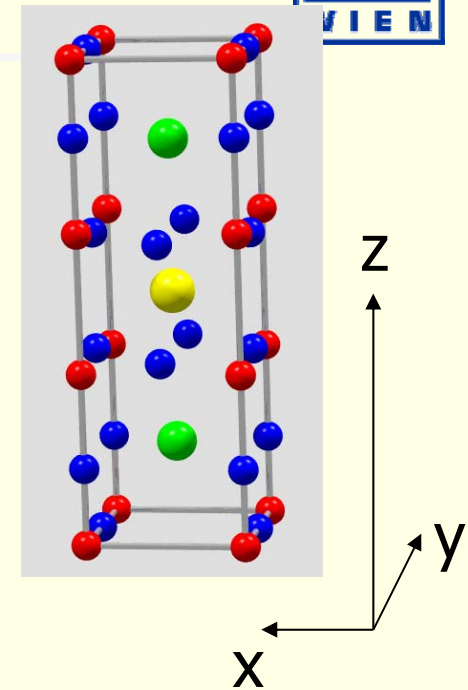
Asymmetry count

EFG (p-contribution)

$$\Delta n_p = p_z - \frac{1}{2}(p_x + p_y)$$

$$V_{zz}^p \propto \Delta n_p \left\langle \frac{1}{r^3} \right\rangle_p$$

EFG is proportional to **asymmetric charge distribution** around given nucleus



partly occupied



Cu partial charges in $\text{YBa}_2\text{Cu}_3\text{O}_7$



	p_x	p_y	p_z	d_{z^2}	$d_{x^2-y^2}$	d_{xy}	d_{xz}	d_{yz}
Cu(1)	0.03	0.07	0.10	1.41	1.65	1.84	1.84	1.86
Cu(2)	0.07	0.07	0.03	1.76	1.44	1.85	1.82	1.82

$$V_{zz}^p \propto \Delta n_{p_z} \left\langle \frac{1}{r^3} \right\rangle_p$$

$$\Delta n_{p_z} = 1/2(p_x + p_y) - p_z$$

$$V_{zz}^d \propto \Delta n_d \left\langle \frac{1}{r^3} \right\rangle_d$$

$$\Delta n_d = (d_{xy} + d_{x^2-y^2}) - 1/2(d_{xz} + d_{yz}) - d_{z^2}$$

0.07e

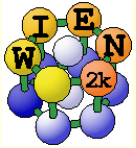
$$V_{zz}^p = 0.038 \times 250 = 9.5 \text{ (} 10^{21} \text{ V/m}^2\text{)}$$

$$V_{zz}^d = -0.288 \times 47 = -13.5$$

a transfer of **0.07 e** into the d_{z^2} would **increase** the **EFG** from **-5.0** by

$$\underline{V_{zz}^d = -0.14 \times 47 = -6.6}$$

bringing it to **-11.6** inclose to the
Experimental value (**-12.3** 10^{21} V/m^2)



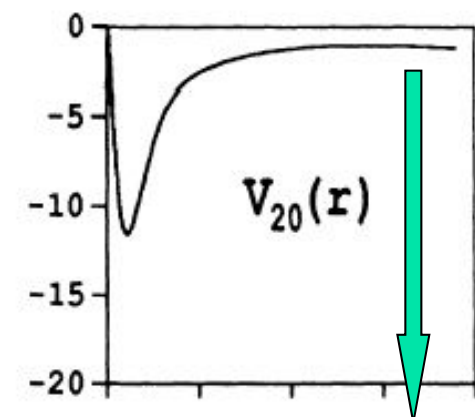
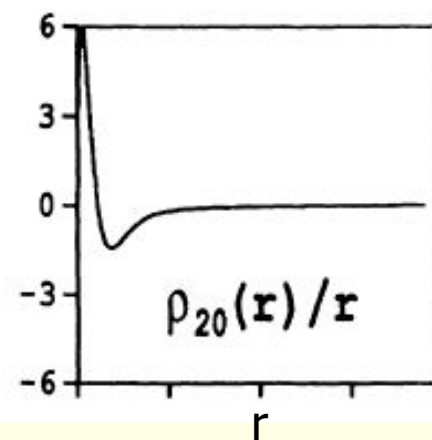
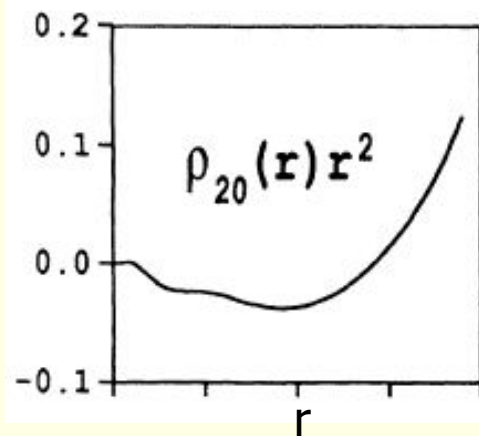
Cu(2) and O(4) EFG as function of r



- EFG is determined by the non-spherical charge density inside sphere

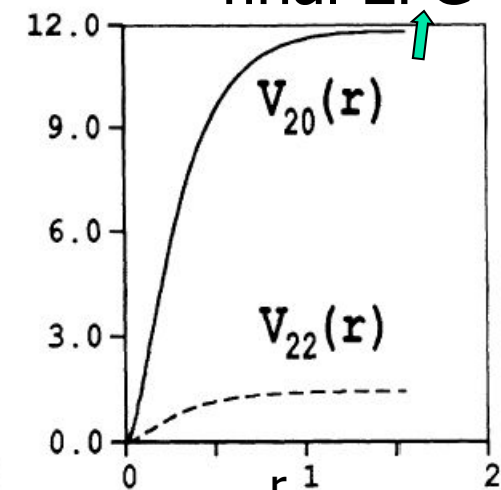
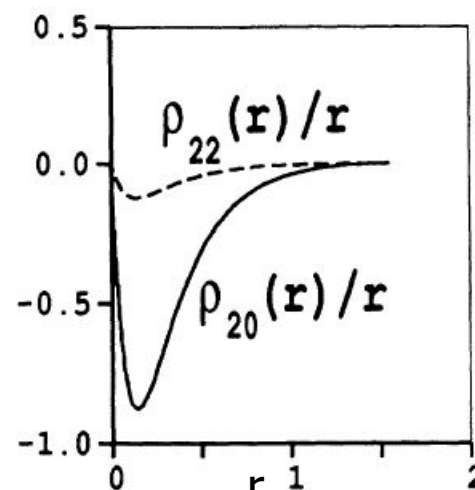
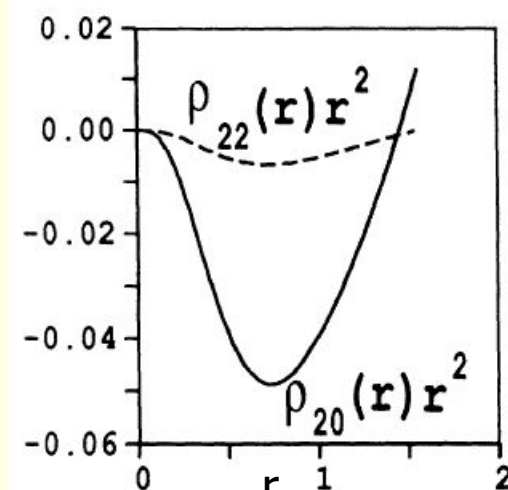
$$\rho(r) = \sum_{LM} \rho_{LM}(r) Y_{LM} \quad V_{zz} \propto \int \frac{\rho(r) Y_{20}}{r^3} dr = \int \rho_{20}(r) r dr$$

- Cu(2)



final EFG

- O(4)

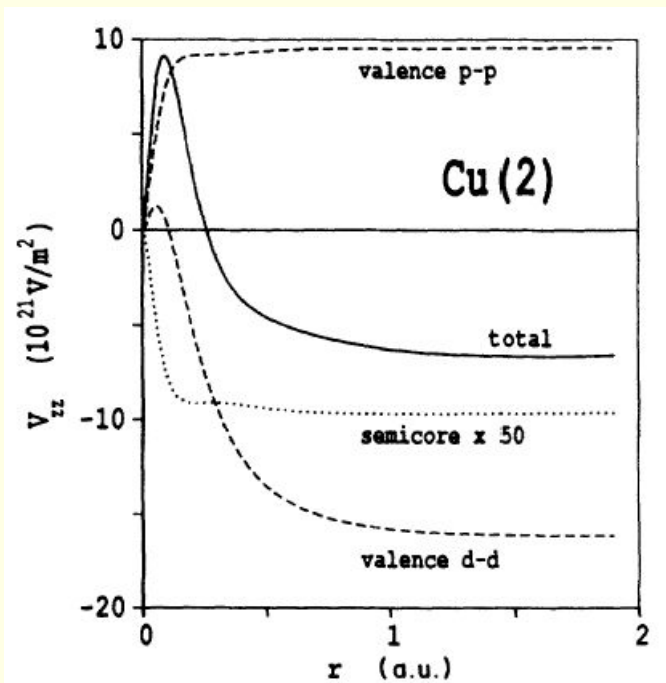




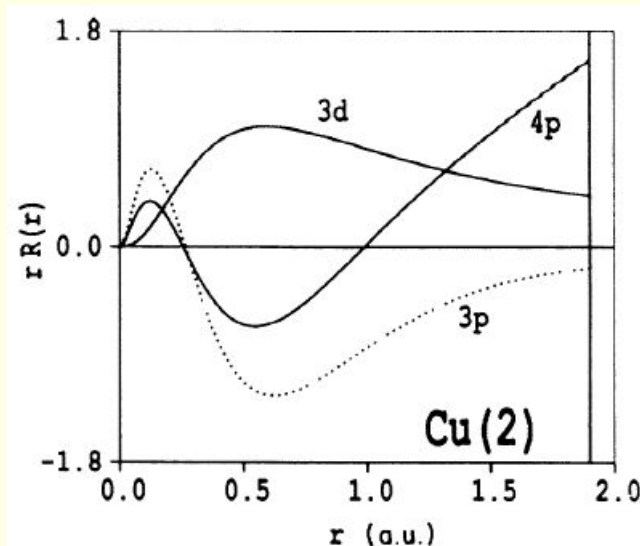
EFG contributions:



- Depending on the atom, the main EFG-contributions come from anisotropies (in occupation or wave function)
 - *semicore p-states* (eg. *Ti 3p* much more important than *Cu 3p*)
 - *valence p-states* (eg. *O 2p* or *Cu 4p*)
 - *valence d-states* (eg. *TM 3d,4d,5d* states; in metals "small")
 - *valence f-states* (only for "localized" 4f,5f systems)



usually only contributions within the first node or within 1 bohr are important.





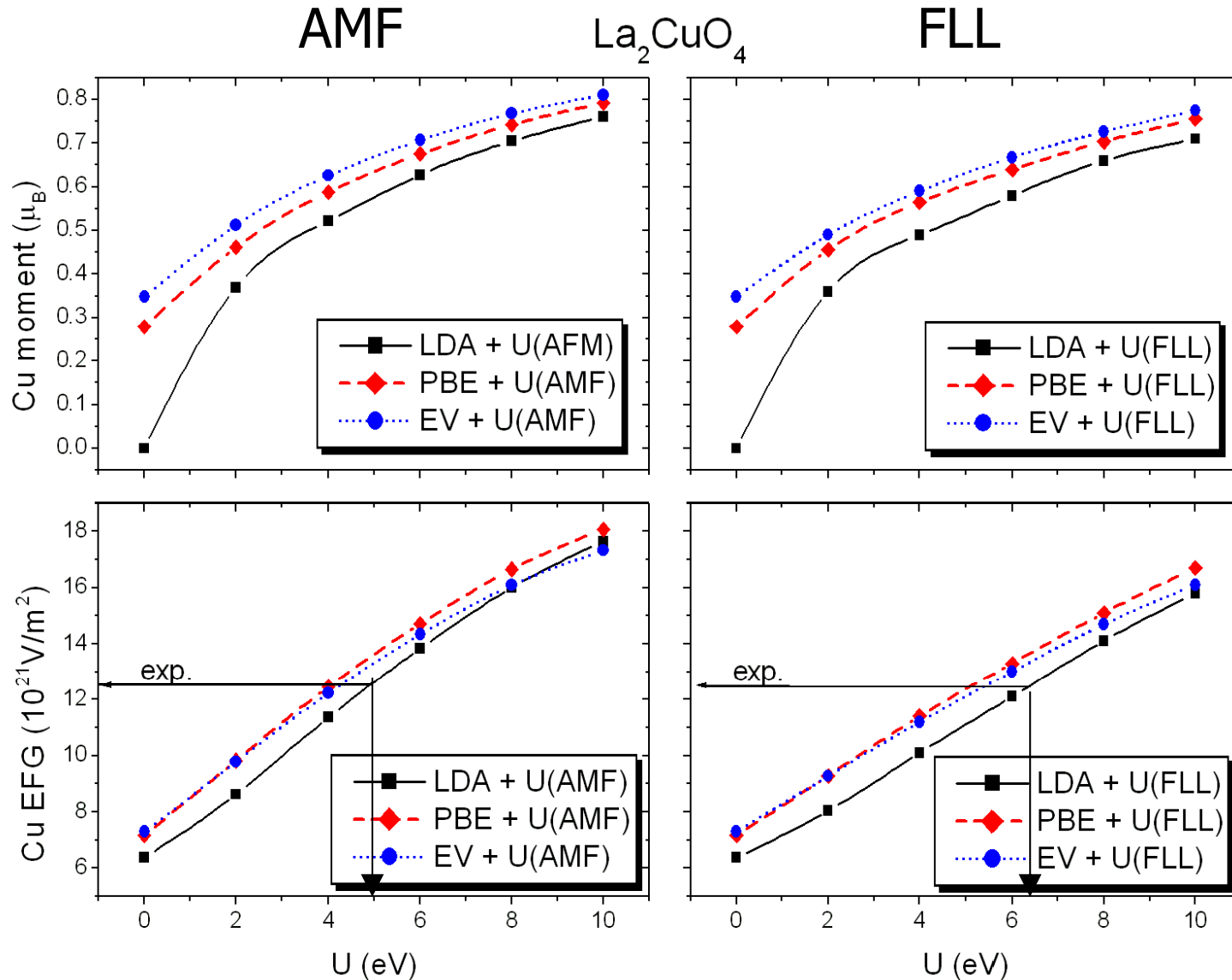
LDA problems in Cuprates



- Undoped Cuprates (La_2CuO_4 , $\text{YBa}_2\text{Cu}_3\text{O}_6$) are nonmagnetic metals instead of antiferromagnetic insulators
- Both, doped and undoped cuprates have a “planar Cu” – EFG which is by a factor of 2-3 too small
- We need a method which gives a better description of correlated 3d electrons: can LDA+U fix these problems ??



Magn. moments and EFG in La_2CuO_4



LDA+U gives AF insulator with reasonable moment

U of 5-6 eV gives exp. EFG

GGAs "mimic" a U of 1-2 eV

(EV-GGA more effective than PBE, but very bad E-tot !!)

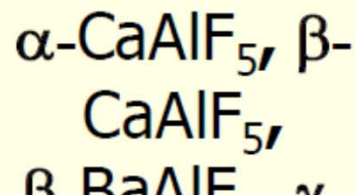
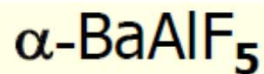
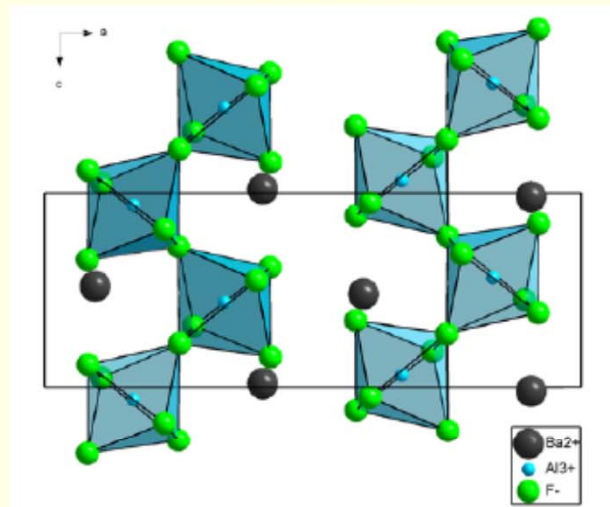


EFGs in fluoroaluminates

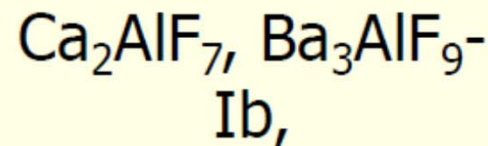
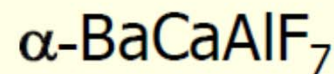
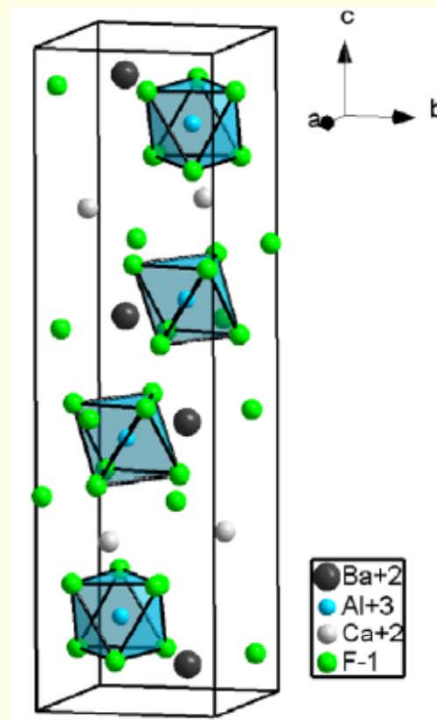


10 different phases of known structures from $\text{CaF}_2\text{-AlF}_3$,
 $\text{BaF}_2\text{-AlF}_3$ binary systems and $\text{CaF}_2\text{-BaF}_2\text{-AlF}_3$ ternary system

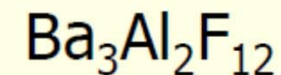
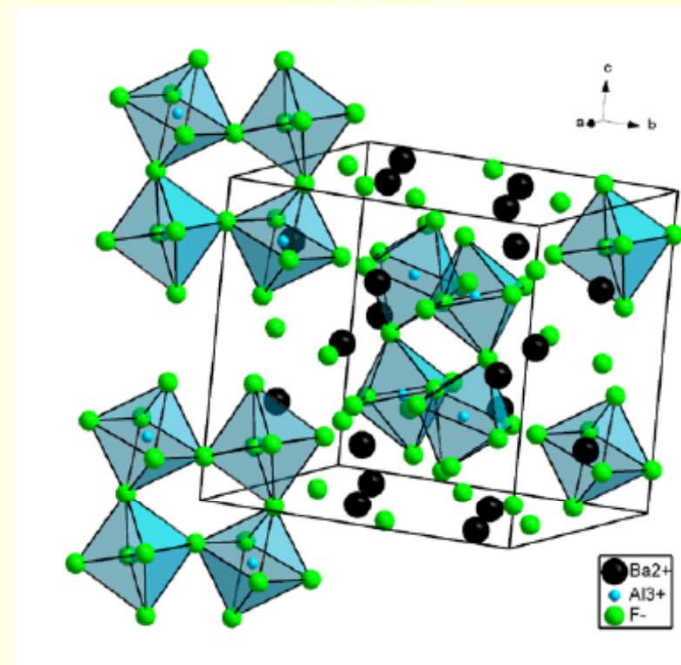
Isolated chains of octahedra linked by corners

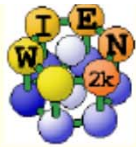


Isolated octahedra

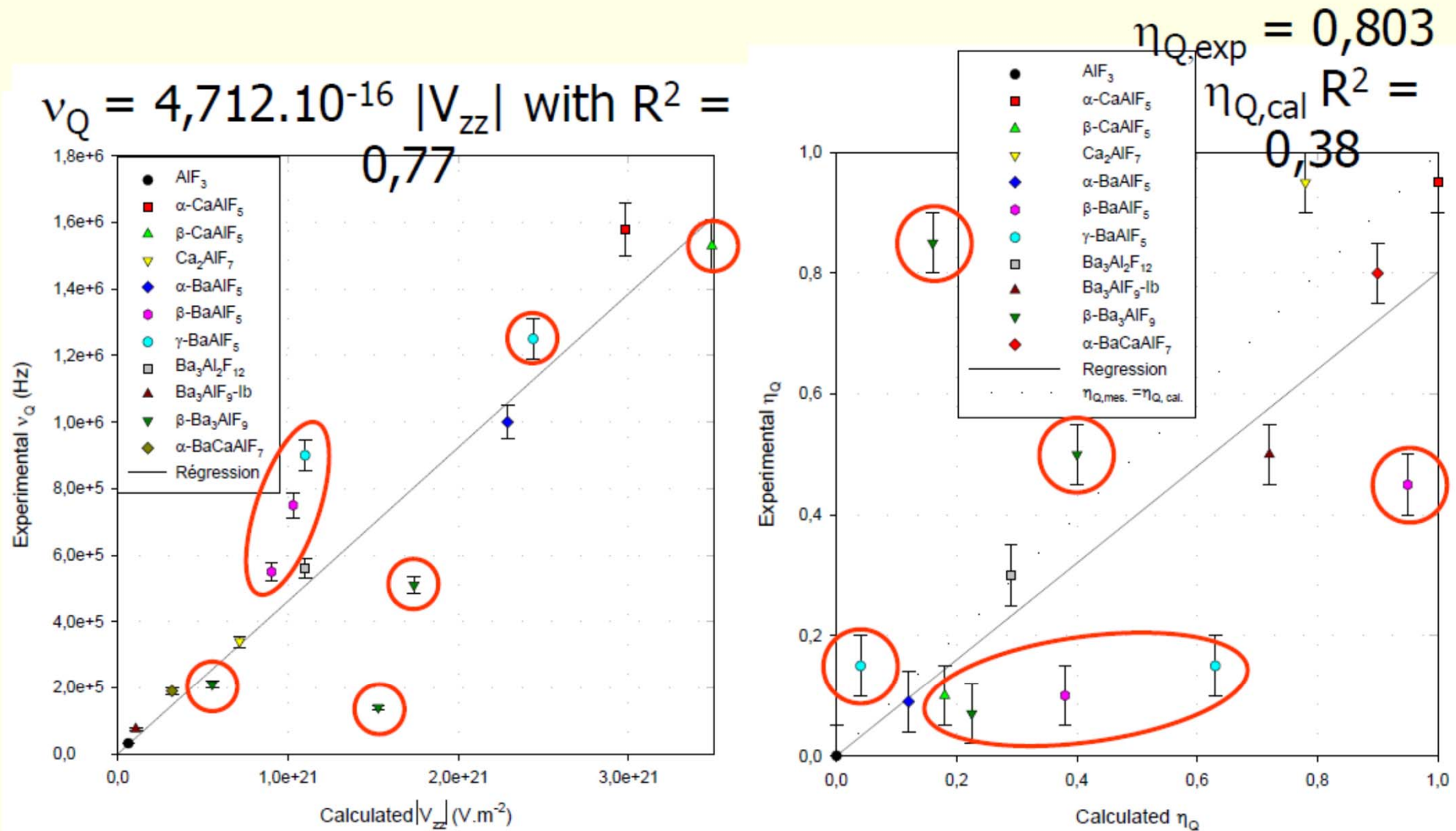


Rings formed by four octahedra sharing corners





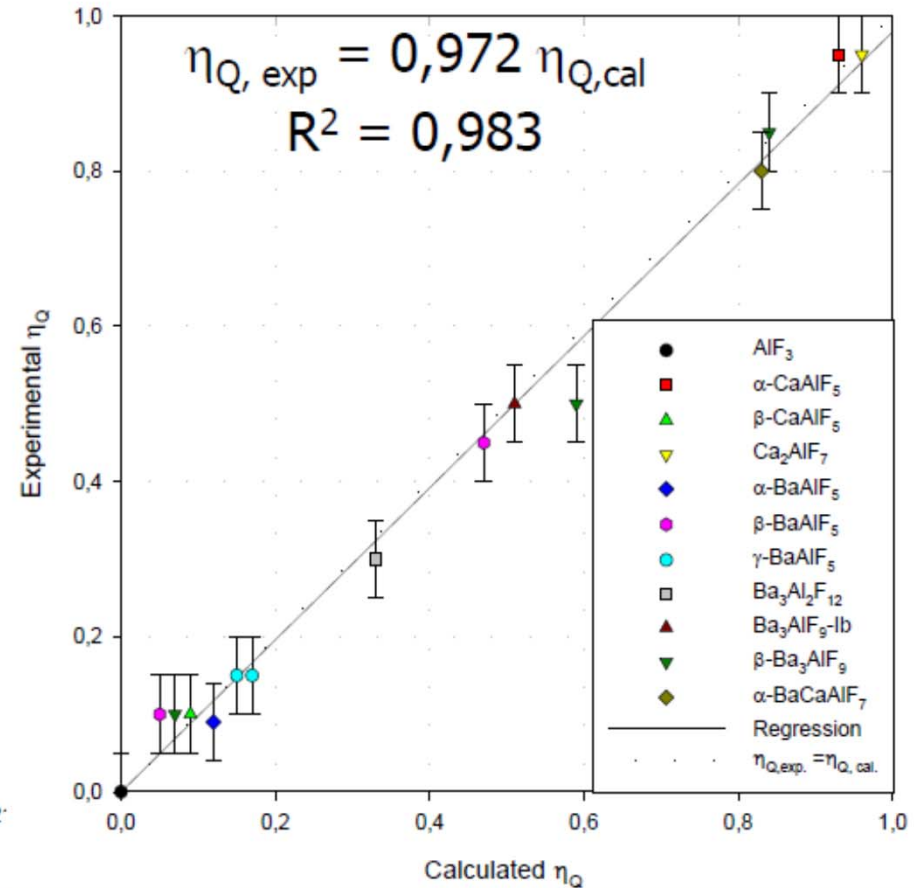
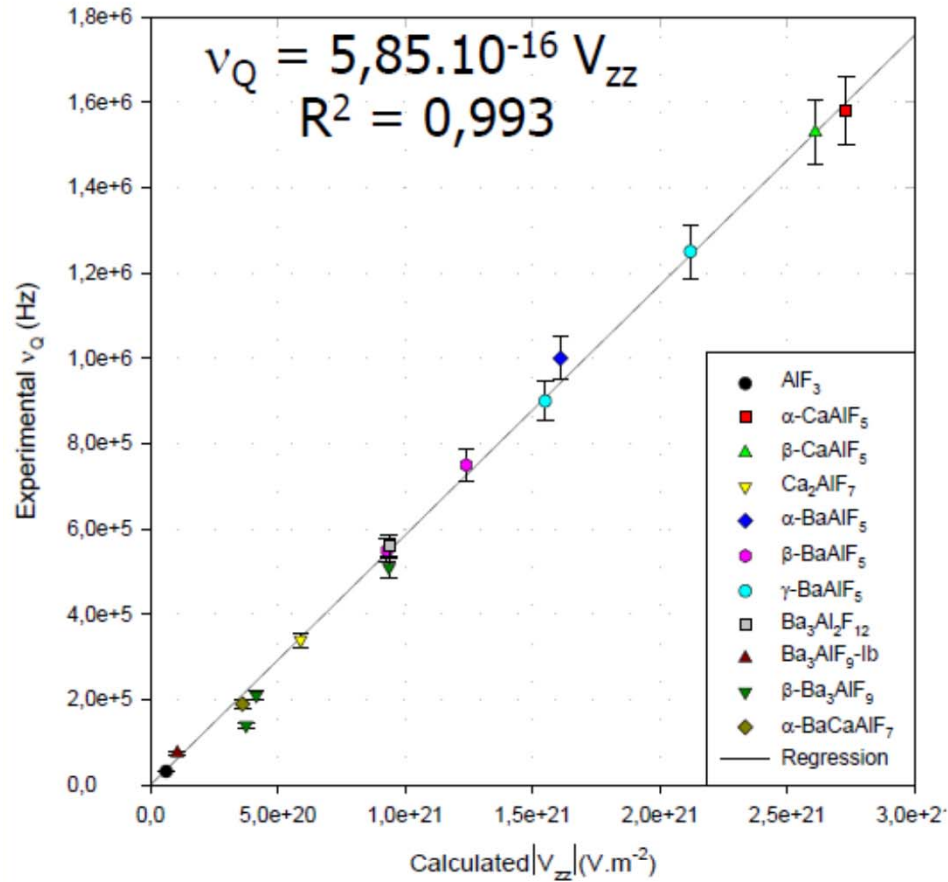
ν_Q and η_Q calculations using XRD data



Important discrepancies when structures are used which were determined from X-ray powder diffraction data



ν_Q and η_Q after structure optimization



Very fine agreement between experimental and calculated values

M.Body, et al., *J.Phys.Chem. A* 2007, 111, 11873
(Univ. LeMans)



Magnetic hyperfine interaction

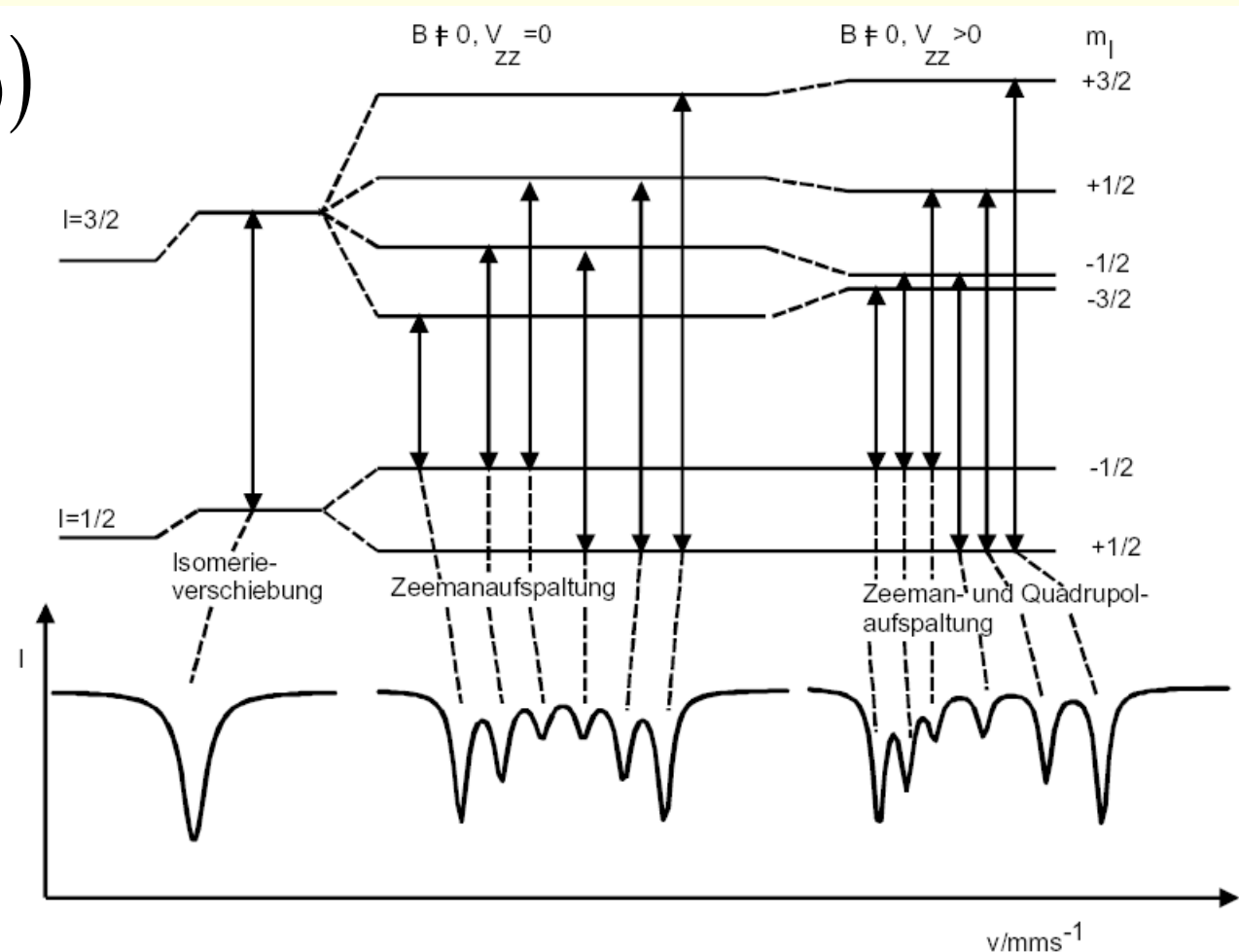


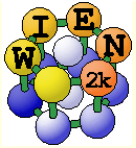
- Zeman - interaction between magnetic moment I of the nucleus and the external magnetic field B (at the nucleus, produced by the spin-polarized e^- in a FM)

$$B \propto (\rho_{\uparrow}(0) - \rho_{\downarrow}(0))$$

B proportional to the **spindensity** at the nucleus

B often proportional to the **magnetic moment** of an atom in a solid.





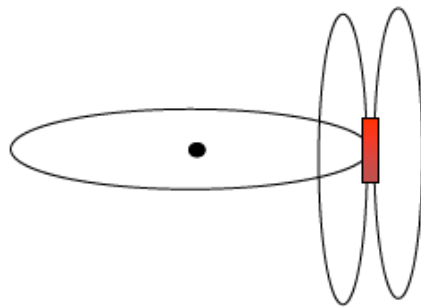
magnetic fields at nucleus:



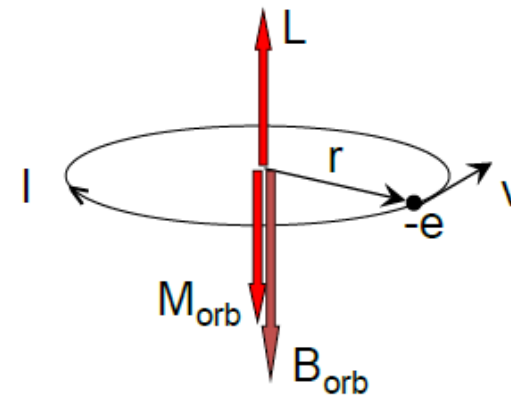
Source of magnetic fields at the nuclear site in an atom/solid

$$B_{\text{tot}} = B_{\text{dip}} + B_{\text{orb}} + B_{\text{fermi}} + B_{\text{lat}}$$

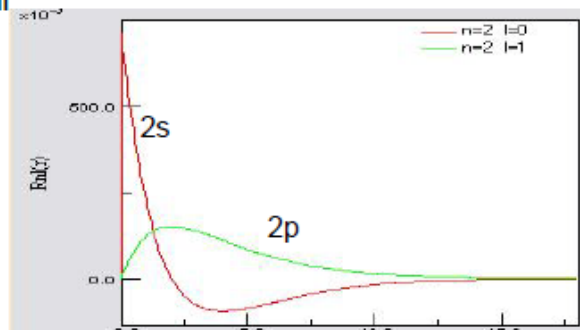
➤ B_{dip} = electron as bar magnet



➤ B_{orb} = electron as current loop

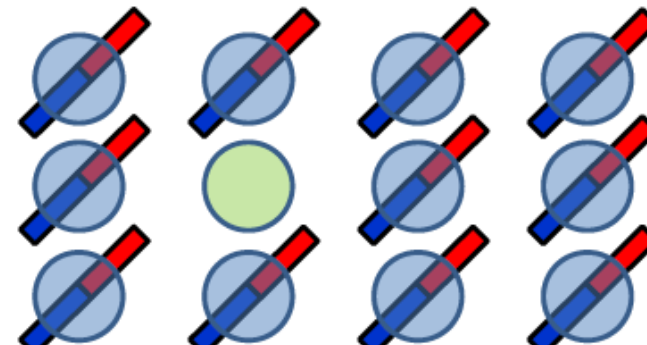


➤ B_{Fermi} = electron in nucleus



$$-\frac{2\mu_B\mu_0}{3} \left(|\psi_{e,\uparrow}(\mathbf{0})|^2 - |\psi_{e,\downarrow}(\mathbf{0})|^2 \right)$$

➤ B_{lat} = neighbours as bar magnets





Magnetic fields at the nucleus:



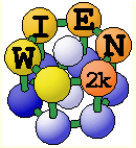
■ Magnetic Hyperfine fields: $B_{\text{tot}} = B_{\text{contact}} + B_{\text{orb}} + B_{\text{dip}}$

■ $B_{\text{contact}} = 8\pi/3 \mu_B [\rho_{\text{up}}(0) - \rho_{\text{dn}}(0)]$... *spin-density at the nucleus*

$$\vec{B}_{\text{orb}} = 2\mu_B \langle \Phi | \frac{S(r)}{r^3} \vec{l} | \Phi \rangle \quad \dots \quad \textit{orbital-moment}$$

$$\vec{B}_{\text{dip}} = 2\mu_B \langle \Phi | \frac{S(r)}{r^3} \left[3(\vec{s} \hat{r}) \hat{r} - \vec{s} \right] | \Phi \rangle \quad \dots \quad \textit{spin-moment}$$

$S(r)$ is reciprocal of the relativistic mass enhancement $S(r) = \left[1 + \frac{\epsilon - V(r)}{2mc^2} \right]^{-1}$



How to do it in WIEN2k:



Magnetic hyperfine field

In regular scf file:

:HFFxxx (Fermi contact contribution)

After post-processing with LAPWDM :

- orbital hyperfine field ("3 3" in case.indmc)
- dipolar hyperfine field ("3 5" in case.indmc)

in case.scfdmup

```
----- top of file: case.indm -----  
-9.          Emin cutoff energy  
1           number of atoms for which density matrix is calculated  
1 1 2      index of 1st atom, number of L's, L1  
0 0        r-index, (l,s)-index  
----- bottom of file -----
```

After post-processing with DIPAN :

- lattice contribution

in case.outputdipan

more info:

UG 7.8 (lapwdm)

UG 8.3 (dipan)

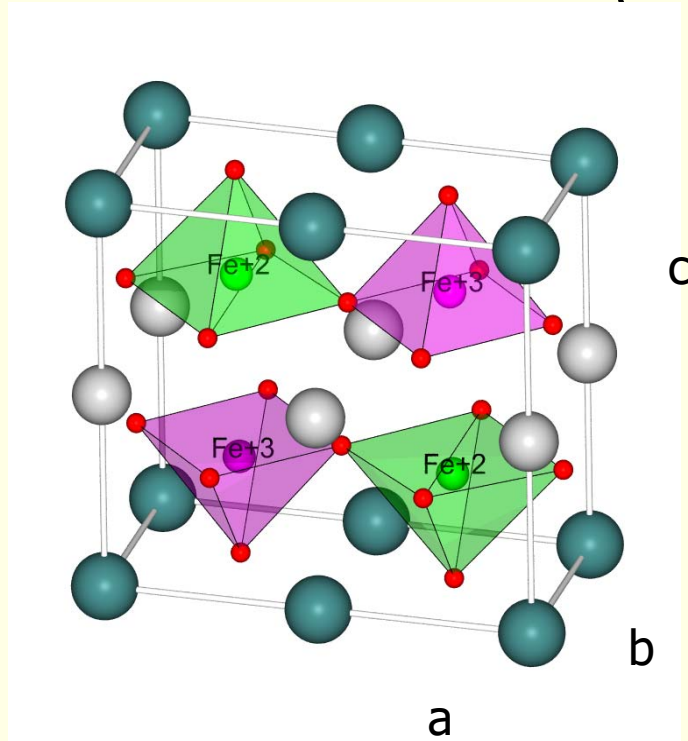


Verwey transition in YBaFe_2O_5



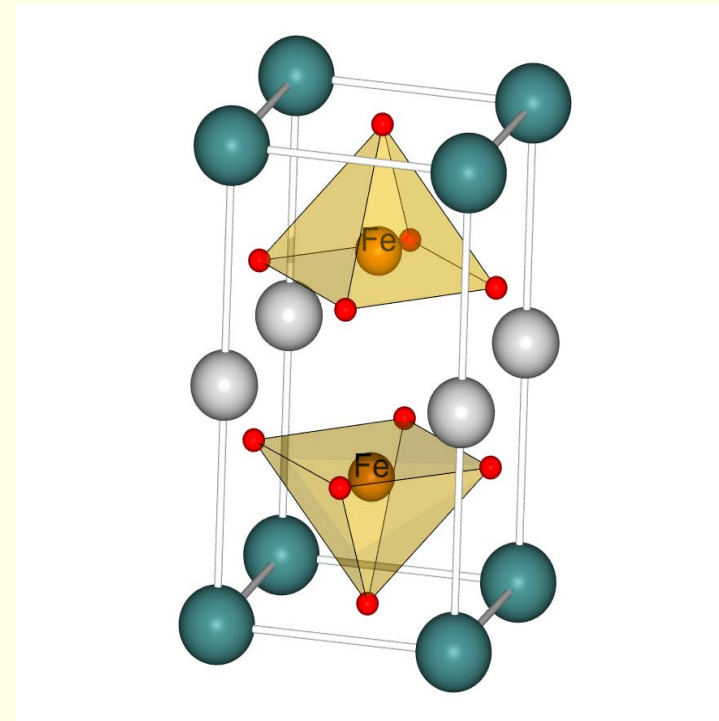
charge ordered (CO) phase:

$Pmma$ $a:b:c=2.09:1:1.96$ (20K)

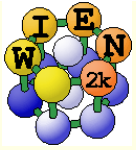


valence mixed (VM) phase:

$Pmmm$ $a:b:c=1.003:1:1.93$ (340K)



- Fe^{2+} and Fe^{3+} form chains along b
- contradicts Anderson charge-ordering conditions with minimal electrostatic repulsion (checkerboard like pattern)
- has to be compensated by orbital ordering and e⁻-lattice coupling



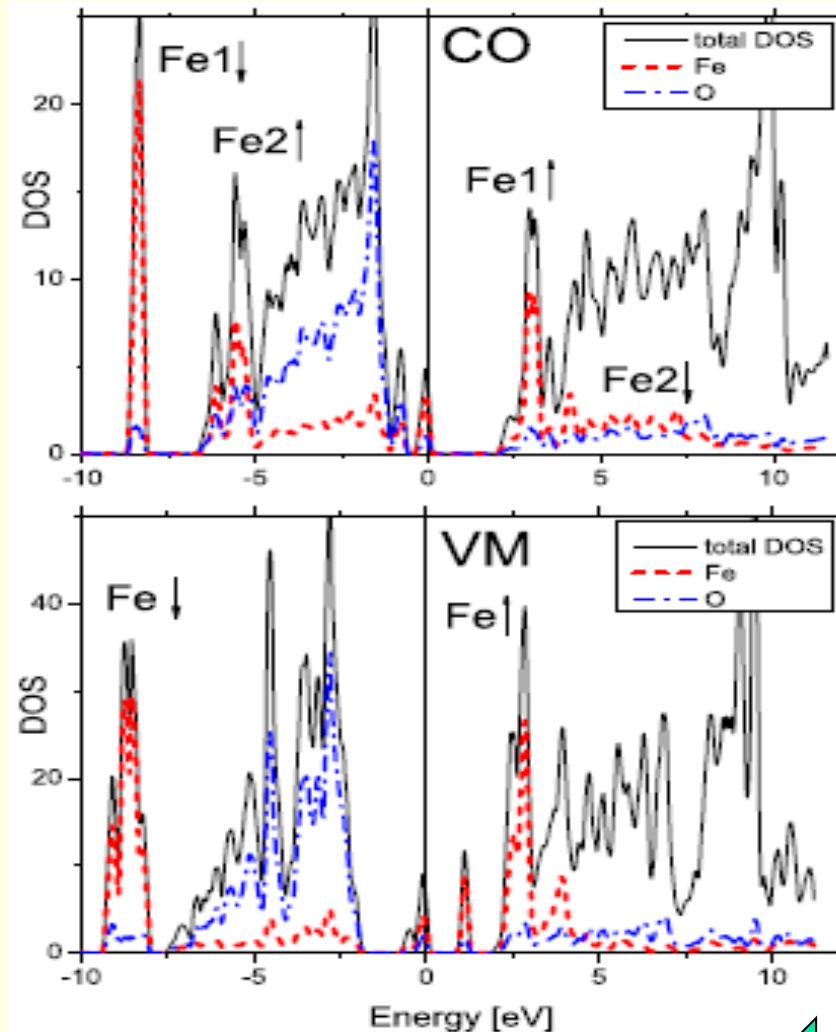
DOS: GGA+U vs. GGA



GGA+U

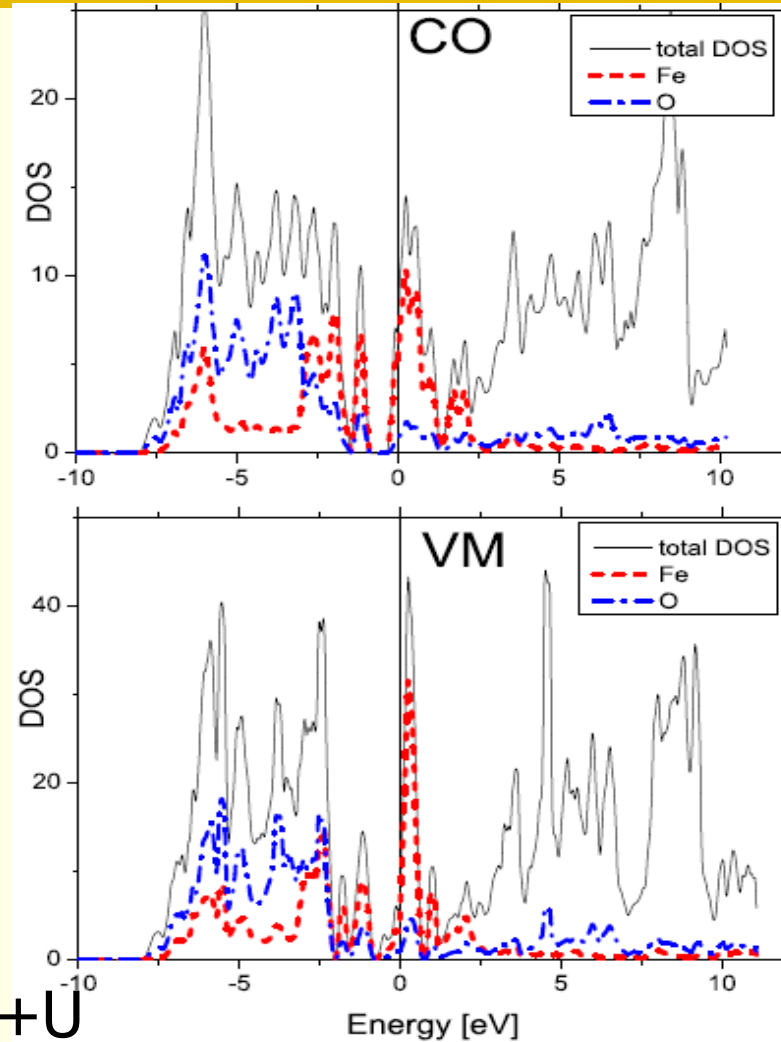
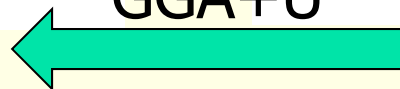
GGA

single lower Hubbard-band in VM splits in CO with Fe³⁺ states lower than Fe²⁺



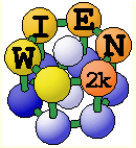
insulator

GGA+U



Energy [eV]

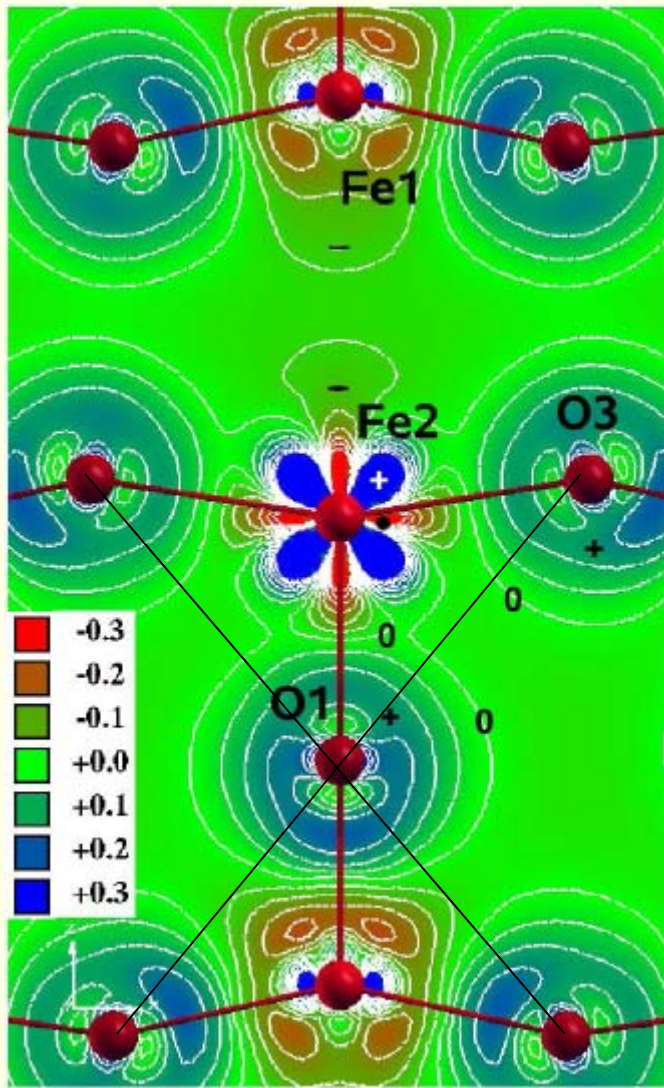
metal



Difference densities $\Delta\rho = \rho_{\text{cryst}} - \rho_{\text{at}}^{\text{sup}}$

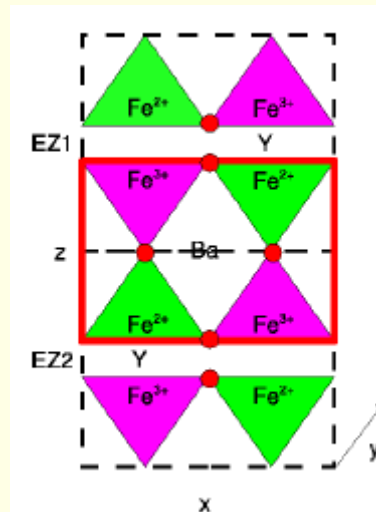


CO phase

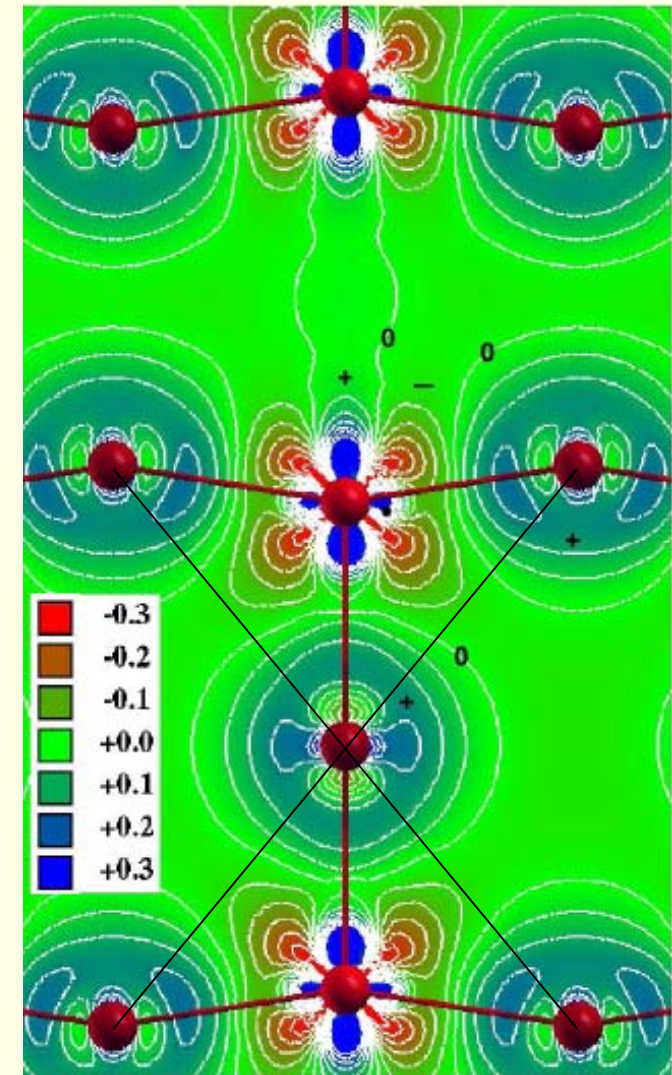


Fe^{2+} : d-xz
 Fe^{3+} : d-x²
 O1 and O3: polarized toward Fe^{3+}

Fe: d-z² Fe-Fe interaction
 O: symmetric



VM phase





Mössbauer spectroscopy



TABLE VIII: Hyperfine fields B (in Tesla), isomer shifts δ (mm/s) and quadrupole coupling constants eQV_{zz} (mm/s) for the CO phase for various exchange and correlation potentials and experiment⁸⁻¹⁰.

CO	U_{eff} [eV]	exp.	GGA+U				LDA	GGA
			5	6	7	8		
Fe ²⁺	B_{dip}	—	-16.29	-16.49	-16.66	-16.83	-6.68	-12.67
	B_{orb}	—	-6.73	-6.90	-8.26	-7.65	-9.57	-6.34
	$B_{contact}$	—	32.25	32.23	32.58	32.60	32.21	31.58
	B_{tot}	~ 8	9.23	8.83	7.66	8.13	15.96	12.57
	δ	~ 1	0.92	0.94	0.96	0.99	0.74	0.79
	eQV_{zz}	$3.6 - 4^a$	3.66	3.74	3.81	3.89	-0.82	2.60
Fe ³⁺	B_{dip}	—	-0.67	-0.60	-0.52	-0.45	1.29	0.39
	B_{orb}	—	-0.52	-0.45	-0.37	-0.28	-7.96	-2.65
	$B_{contact}$	—	37.65	38.28	38.15	37.86	29.64	31.63
	B_{tot}	~ 50	36.46	37.24	37.26	37.12	22.97	29.37
	δ	~ 0.4	0.33	0.30	0.28	0.25	0.50	0.47
	eQV_{zz}	$1 - 1.5^a$	1.46	1.50	1.51	1.52	1.04	-0.30
^a depending on rare earth ion								
VM	U_{eff} [eV]	exp.	GGA+U				LDA	GGA
			5	6	7	8		
Fe ^{2.5+}	B_{dip}	—	-3.00	-2.98	-2.95	-2.87	-2.13	-2.83
	B_{orb}	—	-3.11	-2.99	-2.84	-2.74	-5.47	-4.56
	$B_{contact}$	—	41.17	40.96	41.45	41.17	33.10	36.36
	B_{tot}	~ 30	35.06	34.98	35.67	35.56	25.50	28.98
	δ	~ 0.5	0.53	0.52	0.51	0.49	0.60	0.60
	eQV_{zz}	~ 0.1	0.12	0.13	0.13	0.13	0.19	-0.27



Isomer shift: charge transfer too small in LDA/GGA



TABLE VIII: Hyperfine fields B (in Tesla), isomer shifts δ (mm/s) and quadrupole coupling constants eQV_{zz} (mm/s) for the CO phase for various exchange and correlation potentials and experiment⁸⁻¹⁰.

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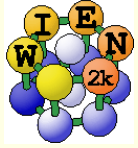


Hyperfine fields: Fe^{2+} has large B_{orb} and B_{dip}



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EFG: Fe²⁺ has too small anisotropy in LDA/GGA



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■ Mössbauer:

- $\Delta = \{e Q V_{zz}(1+\eta^2/3)^{1/2}\} / 2 ;$ $\Delta = (E_\gamma \Delta_v) / c$
- $\Delta_v = (e Q c V_{zz}) / 2 E_\gamma$ $Q(^{57}\text{Fe})=0.16 \text{ b}; E_\gamma =14410 \text{ eV}$
- $V_{zz} [10^{21} \text{ V/m}^2] = 6 * \Delta_v [\text{mm/s}]$

■ NMR:

- $\nu_Q = (3 e Q V_{zz}) / \{2 h I (2 I - 1)\}$ $I .. \text{ nuclear spin quantum } n.$
- $V_{zz} [10^{21} \text{ V/m}^2] = 4.135 \cdot 10^{19} \nu_Q [\text{MHz}] / Q [\text{b}]$ $Q(^{49}\text{Ti})=0.247 \text{ b}$