Methods available in WIEN2k for the treatment of exchange and correlation effects

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Outline of the talk

- Introduction
- Semilocal functionals:
 - GGA and MGGA
 - mBJ potential (for band gap)
 - Input file case.in0
- The DFT-D3 method for dispersion
- On-site methods for strongly correlated electrons:

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- ► DFT+U
- Hybrid functionals
- Hybrid functionals
- ► GW

Total energy in Kohn-Sham DFT¹

$$E_{\text{tot}} = \underbrace{\frac{1}{2} \sum_{i} \int |\nabla \psi_{i}(\mathbf{r})|^{2}}_{T_{s}} d^{3}r + \underbrace{\frac{1}{2} \int \int \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^{3}r d^{3}r'}_{E_{\text{ee}}} + \underbrace{\frac{1}{2} \sum_{\substack{A,B \\ A \neq B}} \frac{Z_{A}Z_{B}}{|\mathbf{R}_{A} - \mathbf{R}_{B}|}}_{E_{\text{nn}}} + E_{\text{xc}}$$

- $T_{\rm s}$: kinetic energy of the non-interacting electrons
- E_{ee} : electron-electron electrostatic Coulomb energy
- *E*_{en} : electron-nucleus electrostatic Coulomb energy
- ▶ *E*_{nn} : nucleus-nucleus electrostatic Coulomb energy
- ► E_{xc} = E_x + E_c : exchange-correlation energy Approximations for E_{xc} have to be used in practice ⇒ The reliability of the results depends mainly on E_{xc}!

¹W. Kohn and L. J. Sham, Phys. Rev. **140**, A1133 (1965)

Approximations for $E_{\rm xc}$ (Jacob's ladder ¹)

 $E_{\mathrm{xc}} = \int \epsilon_{\mathrm{xc}} \left(\mathbf{r} \right) d^3 r$



The accuracy, but also the computational cost, increase when climbing up the ladder

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¹J. P. Perdew *et al.*, J. Chem. Phys. **123**, 062201 (2005)

The Kohn-Sham Schrödinger equations

Minimization of $E_{\rm tot}$ leads to

$$\left(-\frac{1}{2}\nabla^2 + v_{\rm ee}(\mathbf{r}) + v_{\rm en}(\mathbf{r}) + \hat{\mathbf{v}}_{\rm xc}(\mathbf{r})\right)\psi_i(\mathbf{r}) = \epsilon_i\psi_i(\mathbf{r})$$

Two types of \hat{v}_{xc} :

- Multiplicative: $\hat{v}_{xc} = \delta E_{xc} / \delta \rho = v_{xc}$ (KS method)
 - LDA
 - GGA

► Non-multiplicative: $\hat{v}_{xc} = (1/\psi_i)\delta E_{xc}/\delta \psi_i^* = v_{xc,i}$ (generalized KS)

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- Hartree-Fock
- ► LDA+U
- Hybrid (mixing of GGA and Hartree-Fock)
- MGGA
- Self-interaction corrected (Perdew-Zunger)

Semilocal functionals: trends with GGA

$$\epsilon_{\mathrm{xc}}^{\mathrm{GGA}}(
ho,
abla
ho)=\epsilon_{\mathrm{x}}^{\mathrm{LDA}}(
ho)\mathcal{F}_{\mathrm{xc}}(\mathbf{r_s},\mathbf{s})$$

where $F_{\rm xc}$ is the enhancement factor and

$$r_{s} = \frac{1}{\left(\frac{4}{3}\pi\rho\right)^{1/3}} \quad \text{(Wigner-Seitz radius)}$$
$$s = \frac{|\nabla\rho|}{2(3\pi^{2})^{1/3}\rho^{4/3}} \quad \text{(inhomogeneity parameter)}$$
There are two types of GGA:

- Semi-empirical: contain parameters fitted to accurate (i.e., experimental) data.
- Ab initio: All parameters were determined by using mathematical conditions obeyed by the exact functional.

Semilocal functionals: GGA

$$F_{\rm x}(s) = \epsilon_{\rm x}^{
m GGA}/\epsilon_{\rm x}^{
m LDA}$$



Construction of an universal GGA: A failure

Test of functionals on 44 solids¹



¹F. Tran *et al.*, J. Chem. Phys. **144**, 204120 (2016)

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Semilocal functionals: meta-GGA

 $\epsilon_{\rm xc}^{\rm MGGA}(\rho,\nabla\rho,t) = \epsilon_{\rm xc}^{\rm LDA}(\rho) F_{\rm xc}(r_s,s,\alpha)$

where $F_{\rm xc}$ is the enhancement factor and

- $\bullet \ \alpha = \frac{t t_{\rm W}}{t_{\rm TF}}$
 - $\alpha = 1$ where the electron density is uniform
 - $\alpha = 0$ in one- and two-electron regions
 - $\alpha \gg 1$ between closed shell atoms
- \implies MGGA functionals are more flexible

Example: SCAN¹ is

- as good as the best GGA for atomization energies of molecules
- as good as the best GGA for lattice constant of solids

¹J. Sun et al., Phys. Rev. Lett. **115**, 036402 (2015)

Semilocal functionals: meta-GGA

$$F_{\rm x}(s, \alpha) = \epsilon_{\rm x}^{\rm MGGA} / \epsilon_{\rm x}^{\rm LDA}$$



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Semilocal functionals: MGGA_MS2 and SCAN

Test of functionals on 44 solids¹



¹F. Tran et al., J. Chem. Phys. **144**, 204120 (2016)

Semilocal potential for band gap: modified Becke-Johnson

- Standard LDA and GGA functionals underestimate the band gap
- ► Hybrid and *GW* are much more accurate, but also much more expensive
- A cheap alternative is to use the modified Becke-Johnson (mBJ) potential: ¹

$$v_{\mathrm{x}}^{\mathrm{mBJ}}(\mathbf{r}) = c v_{\mathrm{x}}^{\mathrm{BR}}(\mathbf{r}) + (3c-2) \frac{1}{\pi} \sqrt{\frac{5}{6}} \sqrt{\frac{t(\mathbf{r})}{
ho(\mathbf{r})}}$$

where $v_{\rm x}^{\rm BR}$ is the Becke-Roussel potential, t is the kinetic-energy density and c is given by

$$m{c} = lpha + eta \left(rac{1}{V_{ ext{cell}}} \int\limits_{ ext{cell}} rac{|
abla
ho({f r})|}{
ho({f r})} d^3 r
ight)^{m{
ho}}$$

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mBJ is a MGGA potential

¹F. Tran and P. Blaha, Phys. Rev. Lett. **102**, 226401 (2009)

Band gaps with mBJ



How to run a calculation with the mBJ potential?

- 1. init_lapw (choose LDA or PBE)
- 2. init_mbj_lapw (create/modify files)
 - 2.1 automatically done: case.in0 modified and case.inm_vresp created
 - 2.2 run(sp)_lapw -i 1 -NI (creates case.r2v and case.vrespsum)
 - 2.3 save_lapw
- 3. init_mbj_lapw and choose one of the parametrizations:
 - 0: Original mBJ values¹
 - 1: New parametrization²
 - 2: New parametrization for semiconductors²
 - 3: Original BJ potential³

4. run(sp)_lapw ...

¹F. Tran and P. Blaha, Phys. Rev. Lett. **102**, 226401 (2009)

²D. Koller *et al.*, Phys. Rev. B **85**, 155109 (2012)

³A. D. Becke and E. R. Johnson, J. Chem. Phys. 124, 221101 (2006) イロト イヨト イヨト イヨト ヨー クヘベ

Input file case.in0: keywords for the xc-functional

The functional is specified at the 1st line of case.in0. Three different ways:

- 1. Specify a global keyword for E_x , E_c , v_x , v_c :
 - ► TOT XC_NAME
- 2. Specify a keyword for E_x , E_c , v_x , v_c individually:
 - ► TOT EX_NAME1 EC_NAME2 VX_NAME3 VC_NAME4
- 3. Specify keywords to use functionals from LIBXC¹:
 - ► TOT XC_TYPE_X_NAME1 XC_TYPE_C_NAME2
 - ► TOT XC_*TYPE*_XC_*NAME*

where TYPE is the family name: LDA, GGA or MGGA

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http://www.tddft.org/programs/octopus/wiki/index.php/Libxc

¹M. A. L. Marques *et al.*, Comput. Phys. Commun. **183**, 2272 (2012)

Input file case.in0: examples with keywords

 PBE: TOT XC_PBE or TOT EX_PBE EC_PBE VX_PBE VC_PBE or TOT XC_GGA_X_PBE XC_GGA_C_PBE
 mBJ (with LDA for the xc-energy): TOT XC_MBJ

MGGA_MS2: TOT XC_MGGA_MS 0.504 0.14601 4.0 κ,c,b

All available functionals are listed in tables of the UG. and in \$WIENROOT/SRC_lapw0/xc_funcs.h for LIBXC (if installed)

Dispersion methods for DFT

Problem with semilocal functionals:

- They do not include London dispersion interactions
- Results are qualitatively wrong for systems where dispersion plays a major role

Two common dispersion methods for DFT:

Pairwise term¹:

$$E_{\rm c,disp}^{\rm PW} = -\sum_{A < B} \sum_{n=6,8,10,\dots} f_n^{\rm damp}(R_{AB}) \frac{C_n^{AB}}{R_{AB}^n}$$

Nonlocal term²:

$$E_{
m c,disp}^{
m NL} = rac{1}{2} \int \int
ho(\mathbf{r}) \phi(\mathbf{r},\mathbf{r}')
ho(\mathbf{r}') d^3r d^3r'$$

¹S. Grimme, J. Comput. Chem. **25**, 1463 (2004)

²M. Dion et al., Phys. Rev. Lett. **92**, 246401 (2004)

The DFT-D3 method¹ in WIEN2k

► Features of DFT-D3:

- Very cheap (pairwise)
- C_n^{AB} depend on positions of the nuclei (via coordination number)
- Functional-dependent parameters
- Energy and forces (minimization of internal parameters)
- 3-body term
- Installation:
 - Not included in WIEN2k
 - Download and compile the DFTD3 package from http://www.thch.uni-bonn.de/tc/index.php copy the dftd3 executable in \$WIENROOT
 - input file case.indftd3 (if not present a default one is copied automatically)
 - run(sp)_lapw -dftd3 ...
 - case.scfdftd3 is included in case.scf

¹S. Grimme et al., J. Chem. Phys. **132**, 154104 (2010)

The DFT-D3 method: the input file case.indftd3

Default (and recommended) input file:

method	bj	damping function f_n^{damp}
func	default	the one in case. in 0^{\ast}
grad	yes	forces
pbc	yes	periodic boundary conditions
abc	yes	3-body term
cutoff	95	interaction cutoff
cnthr	40	coordination number cutoff
num	no	numerical gradient

*default will work for PBE, PBEsol, BLYP and TPSS. For other functionals, the functional name has to be specified (see dftd3.f of DFTD3 package)

The DFT-D3 method: hexagonal BN¹



¹F. Tran et al., J. Chem. Phys. **144**, 204120 (2016)

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Strongly correlated electrons

Problem with semilocal functionals:

- They give qualitatively wrong results for solids which contain localized 3d or 4f electrons
 - The band gap is too small or even absent like in FeO
 - The magnetic moments are too small
 - Wrong ground state

Why?

The strong on-site correlations are not correctly accounted for by semilocal functionals.

(Partial) solution to the problem:

- Combine semilocal functionals with Hartree-Fock theory:
 - ► DFT+U
 - Hybrid

Even better:

 LDA+DMFT (DMFT codes using WIEN2k orbitals as input exist) On-site DFT+U and hybrid methods in WIEN2k

- For solids, the hybrid functionals are computationally very expensive.
- In WIEN2k the on-site DFT+U¹ and on-site hybrid^{2,3} methods are available. These methods are approximations of the Hartree-Fock/hybrid methods
- ► Applied only inside atomic spheres of selected atoms and electrons of a given angular momentum *l*.

On-site methods \rightarrow As cheap as LDA/GGA.

¹V. I. Anisimov *et al.*, Phys. Rev. B **44**, 943 (1991)

²P. Novák *et al.*, Phys. Stat. Sol. (b) **243**, 563 (2006)

³F. Tran *et al.*, Phys. Rev. B **74**, 155108 (2006)

DFT+U and hybrid exchange-correlation functionals

The exchange-correlation functional is

$$E_{\rm xc}^{\rm DFT+\textit{U/hybrid}} = E_{\rm xc}^{\rm DFT}[\rho] + E^{\rm onsite}[n_{mm'}]$$

where $n_{mm'}$ is the density matrix of the correlated electrons

► For DFT+*U* both exchange and Coulomb are corrected:

$$E^{\text{onsite}} = \underbrace{E_{x}^{\text{HF}} + E_{\text{Coul}}}_{\text{correction}} - \underbrace{E_{x}^{\text{DFT}} - E_{\text{Coul}}^{\text{DFT}}}_{\text{double counting}}$$

There are several versions of the double-counting term

For the hybrid methods only exchange is corrected:

$$E^{\text{onsite}} = \underbrace{\alpha E_{\text{x}}^{\text{HF}}}_{\text{corr.}} - \underbrace{\alpha E_{\text{x}}^{\text{LDA}}}_{\text{d. count.}}$$

where α is a parameter $\in [0, 1]$

How to run DFT+U and on-site hybrid calculations?

- 1. Create the input files:
 - ► case.inorb and case.indm for DFT+U
 - case.ineece for on-site hybrid functionals (case.indm created automatically):

- 2. Run the job (can only be run with runsp_lapw):
 - ► LDA+U: runsp_lapw -orb ...
 - Hybrid: runsp_lapw -eece ...

For a calculation without spin-polarization ($\rho_{\uparrow} = \rho_{\downarrow}$): runsp_c_lapw -orb/eece ...

Input file case.inorb

LDA+U applied to the 4f electrons of atoms No. 2 and 4:

1 2 0	nmod, natorb, ipr
PRATT,1.0	mixmod, amix
2 1 3	iatom, nlorb, lorb
4 1 3	iatom, nlorb, lorb
1	nsic (LDA+U(SIC) used)
0.61 0.07	U J (Ry)
0.61 0.07	U J (Ry)

nsic=0 for the AMF method (less strongly correlated electrons) nsic=1 for the SIC method nsic=2 for the HMF method

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On-site hybrid functional PBE0 applied to the 4f electrons of atoms No. 2 and 4:

-12.0 <mark>2</mark>	emin, natorb
2 1 3	iatom, nlorb, lorb
4 1 3 HYBR	iatom, nlorb, lorb HYBR/EECE
0.25	fraction of exact exchange

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SCF cycle of DFT+U in WIEN2k



Hybrid functionals

- On-site hybrid functionals can be applied only to localized electrons
- Full hybrid functionals are necessary (but expensive) for solids with delocalized electrons (e.g., in *sp*-semiconductors)

Two types of full hybrid functionals available in WIEN2k¹:

unscreened:

$$E_{\mathrm{xc}} = E_{\mathrm{xc}}^{\mathrm{DFT}} + \alpha \left(E_{\mathrm{x}}^{\mathrm{HF}} - E_{\mathrm{x}}^{\mathrm{DFT}} \right)$$

► screened (short-range), $\frac{1}{|\mathbf{r}-\mathbf{r}'|} \rightarrow \frac{e^{-\lambda|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$:

$$\mathbf{E}_{\rm xc} = \mathbf{E}_{\rm xc}^{\rm DFT} + \alpha \left(\mathbf{E}_{\rm x}^{\rm SR-HF} - \mathbf{E}_{\rm x}^{\rm SR-DFT} \right)$$

screening leads to faster convergence with k-points sampling

¹F. Tran and P. Blaha, Phys. Rev. B **83**, 235118 (2011)

Hybrid functionals: technical details

- ▶ 10-1000 times more expensive than LDA/GGA
- **k**-point and MPI parallelization
- Approximations to speed up the calculations:
 - Reduced k-mesh for the HF potential. Example:
 For a calculation with a 12 × 12 × 12 k-mesh, the reduced k-mesh for the HF potential can be:
 6 × 6 × 6, 4 × 4 × 4, 3 × 3 × 3, 2 × 2 × 2 or 1 × 1 × 1

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- Non-self-consistent calculation of the band structure
- Underlying functionals for unscreened and screend hybrid:
 - LDA
 - PBE
 - WC
 - PBEsol
 - B3PW91
 - B3LYP
- Use run_bandplothf_lapw for band structure

Hybrid functionals: input file case.inhf

Example for YS-PBE0 (similar to HSE06 from Heyd, Scuseria and Ernzerhof¹)

0.25	fraction $lpha$ of HF exchange
Т	screened (T, YS-PBE0) or unscreened (F, PBE0)
0.165	screening parameter λ
20	number of bands for the 2nd Hamiltonian
6	GMAX
3	lmax for the expansion of orbitals
3	lmax for the product of two orbitals
1d-3	radial integrals below this value neglected

Important: The computational time will depend strongly on the number of bands, GMAX, Imax and the number of k-points

¹A. V. Krukau et al., J. Chem. Phys. **125**, 224106 (2006)

How to run hybrid functionals?

- $1. \ init_lapw$
- 2. Recommended: run(sp)_lapw for the semilocal functional
- 3. save_lapw
- 4. init_hf_lapw (this will create/modify input files)
 - 4.1 adjust case.inhf according to your needs
 - 4.2 reduced \mathbf{k} -mesh for the HF potential? Yes or no

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- 4.3 specify the \mathbf{k} -mesh
- 5. run(sp)_lapw -hf (-redklist) (-diaghf) ...

SCF cycle of hybrid functionals in WIEN2k



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Calculation of quasiparticle spectra from many-body theory

- In principle the Kohn-Sham eigenvalues should be viewed as mathematical objects and not compared directly to experiment (ionization potential and electron affinity).
- ► The true addition and removal energies e_i are calculated from the equation of motion for the Green function:

$$\left(-\frac{1}{2}\nabla^2 + v_{\rm en}(\mathbf{r}) + v_{\rm H}(\mathbf{r})\right) + \int \Sigma(\mathbf{r},\mathbf{r}',\epsilon_i)\psi_i(\mathbf{r}')d^3r' = \epsilon_i\psi_i(\mathbf{r})$$

► The self-energy ∑ is calculated from Hedin's self-consistent equations¹:

$$\begin{split} \Sigma(1,2) &= i \int G(1,4) W(1^+,3) \Gamma(4,2,3) d(3,4) \\ W(1,2) &= v(1,2) + \int v(4,2) P(3,4) W(1,3) d(3,4) \\ P(1,2) &= -i \int G(2,3) G(4,2) \Gamma(3,4,1) d(3,4) \\ \Gamma(1,2,3) &= \delta(1,2) \delta(1,3) + \int \frac{\delta \Sigma(1,2)}{\delta G(4,5)} G(4,6) G(7,5) \Gamma(6,7,3) d(4,5,6,7) \end{split}$$

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¹L. Hedin, Phys. Rev. **139**, A769 (1965)

The GW and G_0W_0 approximations

• *GW*: vertex function Γ in Σ set to 1:

$$\Sigma(1,2) = i \int G(1,4) W(1^+,3) \Gamma(4,2,3) d(3,4) \approx i G(1,2^+) W(1,2)$$

$$\Sigma(\mathbf{r},\mathbf{r}',\omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} G(\mathbf{r},\mathbf{r}',\omega+\omega') W(\mathbf{r},\mathbf{r}',\omega') e^{-i\delta\omega'} d\omega'$$

$$G(\mathbf{r},\mathbf{r}',\omega) = \sum_{i=1}^{\infty} \frac{\psi_i(\mathbf{r})\psi_i^*(\mathbf{r}')}{\omega - \epsilon_i - i\eta_i} \qquad \qquad W(\mathbf{r},\mathbf{r}',\omega) = \int v(\mathbf{r},\mathbf{r}'')\epsilon^{-1}(\mathbf{r}'',\mathbf{r}',\omega)d^3r''$$

► G₀W₀ (one-shot GW): G and W are calculated using the Kohn-Sham orbitals and eigenvalues. 1st order perturbation theory gives

$$\epsilon_i^{GW} = \epsilon_i^{\mathrm{KS}} + Z(\epsilon_i^{\mathrm{KS}}) \langle \psi_i^{\mathrm{KS}} | \Re(\Sigma(\epsilon_i^{\mathrm{KS}})) - v_{\mathrm{xc}} | \psi_i^{\mathrm{KS}} \rangle$$

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A few remarks on GW

- GW calculations require very large computational ressources
- ► G and W depend on all (occupied and unoccupied) orbitals (up to parameter emax in practice)
- ► GW is the state-of-the-art for the calculation of (inverse) photoemission spectra, but not for optics since excitonic effects are still missing in GW (BSE code from R. Laskowski)

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► *GW* is more accurate for systems with weak correlations

FHI-gap: a LAPW GW code¹

- Based on the FP-LAPW basis set
- Mixed basis set to expand the GW-related quantities
- Interfaced with WIEN2k
- $G_0 W_0$, GW_0 @LDA/GGA(+U)
- Parallelized
- http://www.chem.pku.edu.cn/jianghgroup/codes/fhi-gap.html

¹H. Jiang *et al.*, Comput. Phys. Comput. **184**, 348 (2013)

Flowchart of FHI-gap



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How to run the FHI-gap code?

- 1. Run a WIEN2k SCF calculation (in w2kdir)
- In w2kdir, execute the script gap_init to prepare the input files for GW:

gap_init -d <gwdir> -nkp <nkp> -s 0/1/2 -orb -emax <emax>

- 3. Eventually modify gwdir.ingw
- Execute gap.x or gap-mpi.x in gwdir
- 5. Analyse the results from:
 - 5.1 gwdir.outgw
 - 5.2 the plot of the DOS/band structure generated by gap_analy

Parameters to be converged for a GW calculation

- Usual WIEN2k parameters:
 - Size of the LAPW basis set (*RK*_{max})
 - Number of k-points for the Brillouin zone integrations
- GW-specific parameters:
 - Size of the mixed basis set
 - Number of unoccupied states (emax)
 - ▶ Number of frequencies ω for the calculation of $\Sigma = \int GW d\omega$

Band gaps



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Some recommendations

Before using a method or a functional:

- Read a few papers concerning the method in order to know
 - why it has been used
 - for which properties or types of solids it is supposed to be reliable
 - if it is adapted to your problem
- Do you have enough computational ressources?
 - hybrid functionals and GW require (substantially) more computational ressources (and patience)